Math 141 (Section 5 \& 6)
Prof. Meade
Exam 4
November 25, 2009

University of South Carolina
Fall 2009
Name: $\qquad$
Section: 005 / 006 (circle one)

Instructions:

1. There are a total of 8 problems (including the Extra Credit problem) on 6 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 27 |  |
| 2 | 18 |  |
| 3 | 10 |  |
| 4 | 9 |  |
| 5 | 10 |  |
| 6 | 9 |  |
| 7 | 9 |  |
| 8 | 100 |  |
| Total |  |  |

## Happy Thanksgiving! Beat Clemson!!

1. (27 points) Evaluate each definite integral.
(a) $\int_{0}^{1}\left(\sqrt{t}-t^{2}\right) d t$
(b) $\int_{2}^{4}(3-t)^{9} d t$
(c) $\int_{-\pi / 2}^{\pi / 2} \cos (x) e^{\sin (x)} d x$
2. (18 points) Evaluate each indefinite integral.
(a) $\int\left(2 x-e^{x}\right) d x$
(b) $\int y \sqrt{y+2} d y$
(c) $\int \frac{x+1}{x^{2}+2 x+5} d x$
3. (10 points) Find the function $f$ with $f^{\prime}(t)=2 \cos (t)+\sec ^{2}(t)$ for $-\pi / 2<t<\pi / 2$ and $f(\pi / 3)=4 \sqrt{3}$.
4. (9 points) If $\int_{1}^{5} f(x) d x=12, \int_{4}^{5} f(x) d x=4$ and $\int_{2}^{4} f(x) d x=-2$, find $\int_{1}^{2} f(x) d x$.
5. (10 points) Let $f(x)=\left\{\begin{array}{ll}x+1 & \text { if }-3 \leq x \leq 0 \\ \sqrt{1-x^{2}} & \text { if } 0<x \leq 1\end{array}\right.$. Evaluate $\int_{-3}^{1} f(x) d x$ by interpreting the integral as a difference of areas.
6. (8 points)
(a) Write $\int_{0}^{3}\left(2-x^{2}\right) d x$ as the limit of an appropriate summation.
(b) Evaluate the answer in (a). (Work that involves the Fundamental Theorem of Calculus earns no credit.)
7. (9 points) A storage tank initially contains 5000 liters of water. Water flows into the top of the tank at a rate of $r(t)=200-4 t$ liters per minute, where $0 \leq t \leq 50$ (in minutes). Find the amount of water in the tank after the first 10 minutes.
8. (9 points) Suppose $g(x)=\int_{-1}^{x^{3}} \frac{u^{3}}{u^{2}+1} d u$.
(a) Find $g^{\prime}(x)$.
(b) Find $g(-1)$.
(c) Explain why $g(1)=0$.

Hint: No part of this problem requires any integration, just properties of definite integrals and the FTC.

