Math 141 (Section 5 & 6)
Prof. Meade

University of South Carolina
Fall 20139

Exam 3
November 1, 2013

Name: ______________________
Section: 005 / 006 (circle one)

Instructions:

1. There are a total of 6 problems (including the Extra Credit problem) on 7 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	15	
2	15	
3	20	
4	20	
5	16	
6	14	
Total	100	

Have a Great Weekend!

1. (15 points) Find the absolute maximum and absolute minimum values (including where they occur) for $f(x) = \dfrac{x}{x^2+1}$ on the interval $[0, 3]$.

2. (15 points) Find each limit. Be sure to indicate the indeterminate form each time you use l'Hôpital's Rule.

   (a) $\displaystyle\lim_{x\to 0} \frac{\cos(x) - 1 + \frac{1}{2}x^2}{x^4}$

   (b) $\displaystyle\lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right)$

   (c) $\displaystyle\lim_{x\to 0}(1-2x)^{1/x}$

3. (20 points) Create a sketch of $g(t) = \frac{t^2-4}{t^2-2t}$ that shows the most important aspects of the graph.

   For full credit be sure you address each of the eight guidelines for sketching the most important aspects of a function. (Domain, Intercepts, Symmetry, Asymptotes, Intervals of Increase or Decreases, Local Max and Local Min Values, Concavity and Points of Inflection, Sketch the Graph). If you do not use one of these steps, please indicate why.

4. (20 points) Create a sketch of $f(x) = x\sqrt{2-x^2}$ that shows the most important aspects of the graph.

   For full credit be sure you address each of the eight guidelines for sketching the most important aspects of a function. (Domain, Intercepts, Symmetry, Asymptotes, Intervals of Increase or Decreases, Local Max and Local Min Values, Concavity and Points of Inflection, Sketch the Graph). If you do not use one of these steps, please indicate why.

   Note: $f'(x) = -2\dfrac{x^2-1}{\sqrt{2-x^2}}$ and $f''(x) = \dfrac{2x(x^2-3)}{(2-x^2)^{3/2}}$.

5. (16 points) Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 12 - x^2$.

   (a) What function are you optimizing?

   (b) On what interval?

   (c) Answer the question.

6. (14 points) Suppose that f is a differentiable function with $f(2) = 12$ and $3 \leq f'(x) \leq 5$ for all values of x.

   HINT: Use the Mean Value Theorem on the interval $[2, 8]$, twice.

   (a) Explain how you know that $30 \leq f(8)$.

   (b) How large can $f(8)$ be? That is, find a number M such that $f(8) \leq M$.