MATH 141 (Section 5 & 6)
Prof. Meade

Exam 2
October 9, 2013

University of South Carolina
Fall 2013

Name: Key

Section: 005 / 006 (circle one)

Instructions:

1. There are a total of 6 problems on 4 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	40	
2	16	
3	15	
4	12	
5	12	
6	5	
Total	100	

Good Luck!

1. (40 points) Differentiate each function. (Be sure to label your results.)

(a) $F(x) = 4x^3 - 10x^{2/5} - \pi - 3\ln(x)$

$$F'(x) = 4 \cdot 3x^2 - 10 \cdot \frac{2}{5} x^{-3/5} - 0 - 3 \cdot \frac{1}{x}$$

$$= 12x^2 - 4x^{-3/5} - \frac{3}{x}$$

(b) $y = 2\sqrt{x}\tan(x)$ $= 2x^{1/2}\tan(x)$

$$\frac{dy}{dx} = 2x^{1/2}\sec^2(x) + 2 \cdot \frac{1}{2}x^{-1/2}\tan(x)$$

$$= 2\sqrt{x}\sec^2(x) + \frac{\tan(x)}{\sqrt{x}}$$

(c) $B(\theta) = \dfrac{3^\theta}{\sin(\theta)}$

$$B'(\theta) = \frac{\sin\theta \cdot 3^\theta \ln 3 - 3^\theta \cos\theta}{(\sin\theta)^2}$$

(d) $g(x) = \cos(a^3 + x^3 + e^{3x})$

$$g'(x) = -\sin(a^3 + x^3 + e^{3x}) \cdot (0 + 3x^2 + e^{3x} \cdot 3)$$

$$= -\sin(a^3 + x^3 + e^{3x}) \cdot (3(x^2 + e^{3x}))$$

$$= -3(x^2 + e^{3x})\sin(a^3 + x^3 + e^{3x})$$

(e) $f(x) = \arccos(x) + \tan^{-1}(x^2)$

$$f'(t) = \frac{-1}{\sqrt{1-x^2}} + \frac{1}{(x^2)^2 + 1} \cdot 2x$$

$$= \frac{-1}{\sqrt{1-x^2}} + \frac{2x}{x^4+1}.$$

2. (16 points) The equation of motion of a particle is $s = t^3 - 3t$, where s is in meters and t is in seconds ($t > 0$). Find

(a) the velocity as a function of t

$V = \frac{ds}{dt} = 3t^2 - 3 = 3(t^2-1)$

(b) the acceleration as a function of t

$a = \frac{dv}{dt} = 6t$

(c) the acceleration after 2 s, and

$a(2) = 6 \cdot 2 = 12 \text{ m/sec}^2$

(d) the acceleration when the velocity is 0.

$V = 0 \iff 3(t^2-1) = 0$

$t^2 - 1 = 0$

$t^2 = 1$

$t = \pm$ but you are told that $t > 0$

so only $t = 1$.

When $t = 1$, $a = 6 \cdot 1 = 6 \text{ m/sec}^2$

3. (15 points) Find an equation of the normal line to the curve $x^2 + 4xy = 19 + y^2$ at the point $(2, 3)$. (Note that the equation implicitly defines y as a function of x: $y = y(x)$.)

First, when $x=2, y=3$ we see. $x^2+4xy = 2^2+4\cdot2\cdot3 = 4+24 = 28$

$19+y^2 = 19+3^2 = 19+9 = 28$.

Differentiating wrt x: $\frac{d}{dx}(x^2+4xy(x)) = \frac{d}{dx}(19+y(x)^2)$

$2x + 4x\frac{dy}{dx} + 4y = 2y\frac{dy}{dx}$

When $x=2, y=3$: $2\cdot2 + 4\cdot2\cdot\frac{dy}{dx} + 4\cdot3 = 2\cdot3\frac{dy}{dx}$

$4 + 8\frac{dy}{dx} + 12 = 6\frac{dy}{dx}$

$16 = -2\frac{dy}{dx}$ so $\frac{dy}{dx} = -8$

Normal Line:

$m = -\frac{1}{-8} = \frac{1}{8}$

$y = 3 + \frac{1}{8}(x-2)$

$= \frac{1}{8}x + 3 - \frac{1}{4}$

$= \frac{1}{8}x + \frac{11}{4}$.

4. (12 points) If a snowball melts so that its surface area decreases at a rate of π cm^2/min, find the rate at which the radius is changing when the radius is 10 cm.

$S = 4\pi r^2 \qquad \frac{dS}{dt} = -\pi \qquad$ Find $\frac{dr}{dt}$ when $r = 10$.

$$\frac{dS}{dt} = 4\pi\, 2r \frac{dr}{dt}.$$

$$-\pi = 4\pi \cdot 2 \cdot 10 \frac{dr}{dt}$$

$$-\pi = 80\pi \frac{dr}{dt}$$

$$\frac{dr}{dt} = -\frac{\pi}{80\pi} = -\frac{1}{80} \text{ cm/min.}$$

5. (12 points) Find h' in terms of f' and g' when $h(x) = \dfrac{f(x)g(x)}{f(x) + g(x)}$. (Note: Your answer should be a fraction; be sure to simplify the numerator.)

$$h'(x) = \frac{(f(x)+g(x))(f(x)g'(x)+f'(x)g(x)) - f(x)g(x)(f'(x)+g'(x))}{(f(x)+g(x))^2}$$

$$= \frac{f(x)^2 g'(x) + \cancel{f(x)f'(x)g(x)} + \cancel{f(x)g'(x)g(x)} + f'(x)g(x)^2 - \cancel{f(x)g(x)f'(x)} - \cancel{f(x)g(x)g'(x)}}{(f(x)+g(x))^2}$$

$$= \frac{f(x)^2 g'(x) + f'(x)g(x)^2}{(f(x)+g(x))^2}$$

6. (5 points) Write $|x| = \sqrt{x^2}$ and use the Chain Rule to show that $\dfrac{d}{dx}|x| = \dfrac{x}{|x|}$ for all $x \neq 0$.

$$\frac{d}{dx}|x| = \frac{d}{dx}\sqrt{x^2} = \frac{d}{dx}(x^2)^{1/2}$$

$$= \frac{1}{2}(x^2)^{-1/2} \cdot 2x$$

$$= \frac{2x}{2\sqrt{x^2}}$$

$$= \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}.$$