

$$\text{Given } f(x) = \frac{x^2}{x^2+7}, \quad f'(x) = \frac{14x}{(x^2+7)^2}, \quad f''(x) = \frac{-14(3x^2-7)}{(x^2+7)^3}$$

Find values where $f(x)$ has local max or mins

Where $f(x)$ is increasing or decreasing

Where $f(x)$ is concave up or concave down.

$$f'(x) = 0 \quad \text{when } x = 0$$

$$f'(x) > 0 \quad \text{when } x > 0$$

$$f'(x) < 0 \quad \text{when } x < 0$$

$$f''(x) = 0 \quad \text{when } x = -\sqrt{\frac{7}{3}} \quad \text{and} \quad x = \sqrt{\frac{7}{3}}$$

$$f''(x) > 0 \quad \text{when } -\sqrt{\frac{7}{3}} < x < \sqrt{\frac{7}{3}}$$

$$f''(x) < 0 \quad \text{when } x < -\sqrt{\frac{7}{3}} \quad \text{and} \quad \text{when } x > \sqrt{\frac{7}{3}}$$

So $f(x)$ has a min when $x = 0$

$f(x)$ is concave up $(-\sqrt{\frac{7}{3}}, \sqrt{\frac{7}{3}})$

$f(x)$ is concave down $(-\infty, -\sqrt{\frac{7}{3}}) \cup (\sqrt{\frac{7}{3}}, \infty)$

$f(x)$ is increasing $(0, \infty)$

$f(x)$ is decreasing $(-\infty, 0)$.