

- (a) $f(x) = 2 \cos^2 x - 4 \sin x$, $0 \leq x \leq 2\pi$.
 $f'(x) = -4 \cos x \sin x - 4 \cos x = -4 \cos x (1 + \sin x)$. Note that $1 + \sin x \geq 0$ [since $\sin x \geq -1$], with equality $\Leftrightarrow \sin x = -1 \Leftrightarrow x = \frac{3\pi}{2}$ [since $0 \leq x \leq 2\pi$] $\Rightarrow \cos x = 0$. Thus,
 $f'(x) > 0 \Leftrightarrow \cos x < 0 \Leftrightarrow \frac{\pi}{2} < x < \frac{3\pi}{2}$ and $f'(x) < 0 \Leftrightarrow \cos x > 0 \Leftrightarrow 0 < x < \frac{\pi}{2}$ or $\frac{3\pi}{2} < x < 2\pi$. Thus, f is increasing on $(\frac{\pi}{2}, \frac{3\pi}{2})$ and f is decreasing on $(0, \frac{\pi}{2})$ and $(\frac{3\pi}{2}, 2\pi)$.
- (b) f changes from decreasing to increasing at $x = \frac{\pi}{2}$ and from increasing to decreasing at $x = \frac{3\pi}{2}$. Thus, $f(\frac{\pi}{2}) = -4$ is a local minimum value and $f(\frac{3\pi}{2}) = 4$ is a local maximum value.
- (c) $f''(x) = 4 \sin x (1 + \sin x) - 4 \cos^2 x = 4 \sin x + 4 \sin^2 x - 4(1 - \sin^2 x)$
 $= 8 \sin^2 x + 4 \sin x - 4 = 4(2 \sin x - 1)(\sin x + 1)$
so $f''(x) > 0 \Leftrightarrow \sin x > \frac{1}{2} \Leftrightarrow \frac{\pi}{6} < x < \frac{5\pi}{6}$, and $f''(x) < 0 \Leftrightarrow \sin x < \frac{1}{2}$ and $\sin x \neq -1 \Leftrightarrow 0 < x < \frac{\pi}{6}$ or $\frac{5\pi}{6} < x < \frac{3\pi}{2}$ or $\frac{3\pi}{2} < x < 2\pi$. Thus, f is concave upward on $(\frac{\pi}{6}, \frac{5\pi}{6})$ and concave downward on $(0, \frac{\pi}{6})$, $(\frac{5\pi}{6}, \frac{3\pi}{2})$, and $(\frac{3\pi}{2}, 2\pi)$. There are inflection points at $(\frac{\pi}{6}, -0.50)$ and $(\frac{5\pi}{6}, -0.50)$.