

MATH 141 (Section 3 & 4)
Prof. Meade

University of South Carolina
Fall 2009

Exam 3
October 23, 2009

Name: _____
Section: 003 / 004 (circle one)

Instructions:

1. There are a total of 7 problems (including the Extra Credit problem) on 6 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	30	
2	10	
3	10	
4	10	
5	10	
6	10	
7	20	
Extra Credit	5	
Total	100	

Enjoy Homecoming!

1. (30 points) Find the derivative of each function. Simplify your answer when possible.

(a) $y = x^{10} - \frac{1}{x^2} + \sqrt[5]{x} + e^3$

(b) $g(t) = t^2 \tan(t)$

(c) $y = \frac{1 + \sin(x)}{x + \cos(x)}$

(d) $y = \arctan(e^{\sec(\theta)})$

$$(e) H(z) = \ln \left(\sqrt{\frac{a^2 - z^2}{a^2 + z^2}} \right)$$

HINT: Your final answer should be a rational function.

2. (10 points) Find the second derivative of $f(x) = \frac{x^2}{x^2 + 3}$.

3. (10 points) Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 1.

4. (10 points) The radius of a sphere is decreasing at a rate of 4 mm/s. How fast is the volume changing when the diameter is 80 mm?

NOTE: A sphere with *radius* r has volume $V = \frac{4}{3}\pi r^3$.

5. (10 points) Find the absolute maximum and absolute minimum values of $f(x) = (x^2 - 4)^3$ on the interval $[-1, 2]$.

6. (10 points) Suppose that f is a differentiable function with $f(2) = 12$ and $3 \leq f'(x) \leq 5$ for all values of x . Show that $30 \leq f(8) \leq 42$. (Be sure to show your work.)

HINT: Use the Mean Value Theorem on the interval $[2, 8]$.

7. (20 points) Let $f(x) = 3x^4 - 8x^3 + 6x^2$.

- (a) Find the intervals of increase and decrease.
- (b) Find the intervals of concavity.
- (c) Find the local maximum and local minimum values.
- (d) Find the inflection points

Clearly label each of your answers.

Extra Credit (5 points) Sketch the graph of the function in the previous problem. Be sure to clearly label all extrema and inflection points.

HINT: $f\left(\frac{1}{3}\right) = \frac{11}{27}$