MATH 141 (Section 11 & 12)
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University of South Carolina
Fall 2007

Exam 2 – Practice
October 3, 2007

Name: Key
Section: 011 / 012 (circle one)



Instructions:

1. There are a total of 8 problems on 7 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

| Problem | Points | Score |
|---|---|---|
| 1 | 20 | |
| 2 | 14 | |
| 3 | 8 | |
| 4 | 20 | |
| 5 | 6 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 12 | |
| Total | 100 | |

Study Smart!

1. (20 points) Evaluate the following limits.

(a) $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x^2} \cdot \frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2} = \lim_{x\to 0} \frac{(x^2+4)-4}{x^2(\sqrt{x^2+4}+2)}$

$= \lim_{x\to 0} \frac{x^2}{x^2(\sqrt{x^2+4}+2)} = \lim_{x\to 0} \frac{1}{\sqrt{x^2+4}+2} = \frac{1}{2+2} = \underline{\underline{\frac{1}{4}}}$

(b) $\lim_{x\to 1} \frac{x^3-x^2}{x-1} = \lim_{x\to 1} \frac{x^2(x-1)}{x-1} = \lim_{x\to 1} x^2 = \underline{\underline{1}}$

(c) $\lim_{x\to\infty} \frac{2x-7}{4x-x^2} = \lim_{x\to\infty} \frac{x(2-7/x)}{x^2(\frac{4}{x}-1)} = \lim_{x\to\infty} \frac{1}{x} \cdot \lim_{x\to\infty} \frac{2-7/x}{\frac{4}{x}-1}$

$= 0 \cdot \frac{2}{-1} = \underline{\underline{0}}$

(d) $\lim_{x\to 0} \frac{x\sin(x)}{1-\cos(x)} \cdot \frac{1+\cos(x)}{1+\cos(x)} = \lim_{x\to 0} \frac{x\sin(x)\,(1+\cos(x))}{1-\cos^2(x)}$

$= \lim_{x\to 0} \frac{x\,\cancel{\sin(x)}\,(1+\cos(x))}{\sin^{\cancel{2}}(x)} = \lim_{x\to 0} \frac{x}{\sin(x)}(1+\cos(x))$

$= \lim_{x\to 0} \frac{x}{\sin(x)} \lim_{x\to 0} (1+\cos(x)) = 1 \cdot (1+1) = \underline{\underline{2}}$

2. (14 points) Find $\lim_{x \to a} f(x)$ where $f(x) = \sqrt{x-5}$ for

$(x \geq 5)$

(a) $a = 0$

dne.

(b) $a = 5^+$

$0$

(c) $a = -5^-$

dne

(d) $a = 5$

dne

(e) $a = -5$

dne.

(f) $a = \infty$

$\infty$

(g) $a = -\infty$

dne

3. (8 points) Find all points where $f(x) = \dfrac{x+3}{|x^2+3x|}$ is *not* continuous.

$$f(x) = \frac{x+3}{|x(x+3)|} = \frac{x+3}{|x|\,|x+3|} \quad \text{for } x \neq 0 \text{ \& } x = -3.$$

For all other values of x, f(x) is defined and continuous because both the numerator and denominator are continuous (and the denominator is not zero). This function is <u>not</u> continuous for <u>x = 0</u> and <u>x = -3</u>.

4. (20 points) Find the derivative of each of the following functions.

(a) $f(x) = x^8 - 3\sqrt{x} + 5x^{-3}$ $= x^8 - 3x^{1/2} + 5x^{-3}$

$f'(x) = 8x^7 - \frac{3}{2}x^{-1/2} - 15x^{-4}$

(b) $f(x) = x^2 \sin(x)$

$f'(x) = x^2 \cos(x) + 2x \sin(x)$

(c) $f(x) = \dfrac{\sin(x)}{2x + \cos(x)}$

$$f'(x) = \frac{(2x+\cos(x))\cos(x) - \sin(x)(2-\sin(x))}{(2x+\cos(x))^2}$$

$$= \frac{2x\cos(x) + \cos^2(x) - 2\sin(x) + \sin^2(x)}{(2x+\cos(x))^2}$$

$$= \frac{2x\cos(x) - 2\sin(x) + 1}{(2x+\cos(x))^2}$$

5. (6 points) The differentiable functions $f$ and $g$ have $f(1) = 1$, $g(1) = 2$, $f'(1) = 3$, and $g'(1) = -1$. Evaluate each of the following expressions.

(a) $\frac{d}{dx}(f(x)g(x))\big|_{x=1}$ $= f(1)g'(1) + f'(1)g(1)$

$= 1(-1) + (3)(2)$

$= -1 + 6$

$\underline{\underline{= 5}}$

(b) $\frac{d}{dx}\sqrt{f(x)}\Big|_{x=1}$ $= \frac{d}{dx}(f(x))^{1/2}\Big|_{x=1} = \frac{1}{2}f(x)^{-1/2}f'(x)\Big|_{x=1}$

$= \frac{1}{2}f(1)^{-1/2}f'(1)$

$= \frac{1}{2}(1)^{-1/2}(3) = \underline{\underline{\frac{3}{2}}}$

6. (10 points) The amount of water in a tank $t$ minutes after it has started to drain is given by $W = 100(t-15)^2$. gallons.

$W = 100(t^2 - 30t + 225)$

(a) What is the average rate at which water flows out of the tank during the first 5 minutes?

$W_{ave} = \frac{W(5) - W(0)}{5 - 0} = \frac{100(-10)^2 - 100(-15)^2}{5}$

$= \frac{100(100) - 100(225)}{5} = \frac{100}{5}(100 - 225) = 20(-125)$

$= -2500$ gallons per minute.

(b) What is the instantaneous rate at which water flows out of the tank at the end of 5 minutes?

$W_{inst} = \frac{dW}{dt}\Big|_{t=5} = 100(2t - 30)\Big|_{t=5}$

$= 100(10 - 30)$

$= \underline{\underline{-2000}}$ gallons per minute

7. (10 points)

(a) State the definition of the derivative of a function $f(x)$ in terms of a limit.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

or

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

(b) Use the definition of the derivative to find $dy/dx$ for $y = \sqrt{9 - 4x}$.

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{x \to 0} \frac{\sqrt{9-4x-4h} - \sqrt{9-4x}}{h} \cdot \frac{\sqrt{9-4x-4h} + \sqrt{9-4x}}{\sqrt{9-4x-4h} + \sqrt{9-4x}}$$

$$= \lim_{x \to 0} \frac{(9-4x-4h) - (9-4x)}{h\left(\sqrt{9-4x-4h} + \sqrt{9-4x}\right)}$$

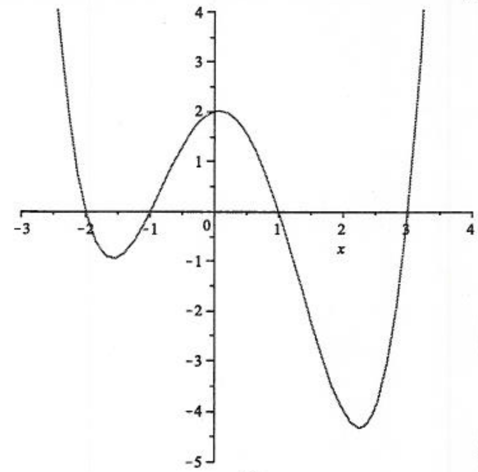
$$= \lim_{x \to 0} \frac{-4h}{h\left(\sqrt{9-4x-4h} + \sqrt{9-4x}\right)}$$

$$= \lim_{h \to 0} \frac{-4}{\sqrt{9-4x-4h} + \sqrt{9-4x}}$$

$$= \frac{-4}{\sqrt{9-4x} + \sqrt{9-4x}}$$

$$= \frac{-4}{2\sqrt{9-4x}} = -2/\sqrt{9-4x}.$$

8. (12 points) The figure below shows the graph of $y = f'(x)$ for an unspecified function $f$.



(a) For what value of $x$ does the curve $y = f(x)$ have a horizontal tangent line?

$x = -2$

$x = -1$

$x = 1$

$x = 3$

Where $f'(x) = 0$.

(b) Over what intervals does the curve $y = f(x)$ have tangent lines with positive slope?

$x < -2$

$-1 < x < 1$

$x > 3$

Where $f'(x) > 0$.

(c) Over what intervals does the curve $y = f(x)$ have tangent lines with negative slope?

$-2 < x < -1$

$1 < x < 3$.

Where $f'(x) < 0$.