MATH 141 (Section 11 & 12)
Prof. Meade

University of South Carolina
Fall 2007

Exam 1
September 13, 2007

Name: Key
SS #: __________

Instructions:

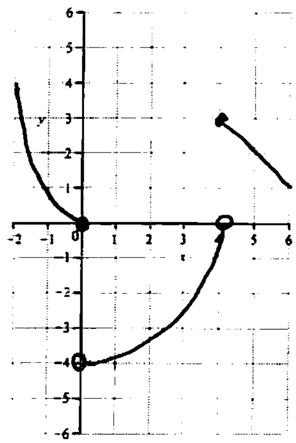
1. There are a total of 8 problems on 6 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	10	
2	15	
3	18	
4	12	
5	7	
6	18	
7	6	
8	14	
Total	100	

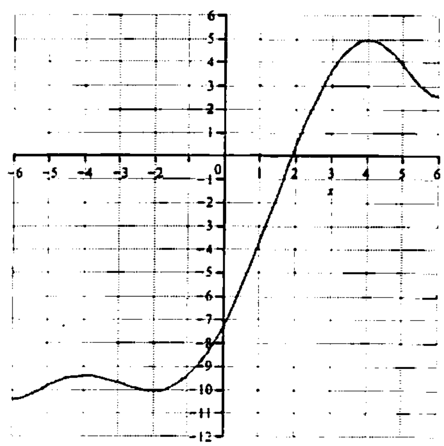
Good Luck!

1. (10 points) Use the axes provided to sketch the graph of the function

$$f(x) = \begin{cases} x^2 & x \leq 0 \\ -\sqrt{16 - x^2} & 0 < x < 4 \\ 7 - x & x \geq 4 \end{cases}$$



2. (15 points) Use the graph of the function f in the figure below to answer the following questions.



(a) Find $f(-2)$. $f(-2) = -10$

(b) What is the maximum value of the function f? max. value is 5

(c) For what value of x does f reach its maximum value? & occurs at $x = 4$

(d) What is the largest interval containing $x = 0$ on which f is invertible?

The graph passes the Horizontal Line Test for $-2 \leq x \leq 4$

(e) What is $f^{-1}(-4)$?

$f^{-1}(-4) = x$ when $-4 = f(x)$ This gives us $x = 1 = f^{-1}(-4)$

3. (18 points) Let $f(x) = \sqrt{x-3}$ and $g(x) = \sqrt{x^2+3}$.

(a) What is the domain of f? $x-3 \geq 0$ when $x \geq 3$

domain of f is $x \geq 3$, or $x \in [3, \infty)$

(b) What is the domain of g? $x^2+3 \geq 3 > 0$ for all real numbers x

domain of g is $-\infty < x < \infty$ or $x \in (-\infty, \infty)$

(c) Find a formula for $\frac{g}{f}$.

$$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{x^2+3}}{\sqrt{x-3}} = \sqrt{\frac{x^2+3}{x-3}}$$

(d) What is the domain of $\frac{g}{f}$?

domain of $\frac{g}{f}$ is all x in domain of f & domain of g with $f(x) \neq 0$

so domain of $\frac{g}{f}$ is $x > 3$ or $x \in (3, \infty)$

(e) Find a formula for $g \circ f$.

$$(g \circ f)(x) = \sqrt{f(x)^2 + 3} = \sqrt{\left(\sqrt{x-3}\right)^2 + 3} = \sqrt{(x-3)+3} = \sqrt{x}$$

(f) What is the domain of $g \circ f$?

domain of $g \circ f$ is all x in domain of f with $f(x)$ in domain of g.

This gives domain of $g \circ f$ is $x \geq 3$ or $x \in [3, \infty)$.

4. (12 points) Determine if each of the following functions is even, odd, or neither.

   (a) $f(x) = \sin(x) + \cos(x)$

   $f(-x) = \sin(-x) + \cos(-x) = -\sin(x) + \cos(x)$ neither

   (b) $g(x) = e^{x^2+2}$

   $g(-x) = e^{(-x)^2+2} = e^{x^2+2} = g(x)$ even

   (c) $h(x) = \sin^2(x) + \cos(x)$

   $h(-x) = (\sin(-x))^2 + \cos(-x) = (-\sin(x))^2 + \cos(x) = \sin^2(x) + \cos(x) = h(x)$

   even

   (d) $k(x) = x^{13} + 12x^{11} - 10x^5 + \pi x^3 - \frac{1}{2}$

   $k(-x) = (-x)^{13} + 12(-x)^{11} - 10(-x)^5 + \pi(-x)^3 - \frac{1}{2}$
   $= -x^{13} - 12x^{11} + 10x^5 - \pi x^3 - \frac{1}{2}$ neither

5. (7 points) Let $f(x) = \dfrac{1-2x}{1+x}$ for $x > -1$. Find a formula for $f^{-1}(x)$, or explain why the inverse does not exist.

$y = f(x) = \frac{1-2x}{1+x}$

$(1+x)y = 1-2x$

$y + xy = 1-2x$

$2x + xy = 1-y$

$(2+y)x = 1-y$

$x = \frac{1-y}{2+y}$

$y = \frac{1-x}{2+x}$ so $f^{-1}(x) = \frac{1-x}{2+x}$.

6. (18 points)

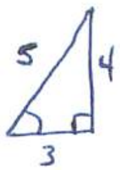
(a) Find the exact numerical value for

$$\cos\left(\arcsin\left(\frac{4}{5}\right)+\arctan\left(\frac{4}{3}\right)\right).$$

Use: $\cos(\theta+\varphi) = \cos\theta\cos\varphi - \sin\theta\sin\varphi$

with $\theta = \arcsin\left(\frac{4}{5}\right)$ & $\varphi = \arctan\left(\frac{4}{3}\right)$.

So $\sin\theta = \frac{4}{5}$ and $\tan\varphi = \frac{4}{3}$



Note that $\theta = \varphi$.

Now:

$\cos\left(\arcsin\left(\frac{4}{5}\right) + \arctan\left(\frac{4}{3}\right)\right) = \cos(\theta+\varphi)$

$= \cos(\theta)\cos(\varphi) - \sin(\theta)\sin(\varphi)$

$= \frac{3}{5}\cdot\frac{3}{5} - \frac{4}{5}\cdot\frac{4}{5}$

$= \frac{9}{25} - \frac{16}{25}$

$= -\frac{7}{25}$

(b) Express the following function as a rational function of x:

$$f(x) = \frac{\ln(e^x) - \ln(e^2)}{e^{\ln(x^2+1)} - e^{2\ln(x)}}.$$

$$f(x) = \frac{\ln(e^x) - \ln(e^2)}{e^{\ln(x^2+1)} - e^{2\ln(x)}} = \frac{x-2}{x^2+1-e^{\ln(x^2)}} = \frac{x-2}{(x^2+1)-x^2} = \frac{x-2}{1} = \underline{\underline{x-2}}$$

(c) Consider the parametric curve given by $x = 2\cos(t)$, $y = 4\sin(t)$ $(0 \le t \le 2\pi)$. Eliminate the parameter t and find y as a function of x.

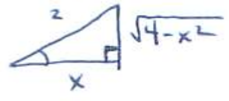
HINT: Simplify your answer until it has no trigonometric functions.

Long Sol'n: $x = 2\cos(t)$

$\frac{x}{2} = \cos(t)$

$t = \arccos\left(\frac{x}{2}\right)$.

Then $y = 4\sin(t) = 4\sin\left(\arccos\left(\frac{x}{2}\right)\right)$



$= 4\,\frac{\sqrt{4-x^2}}{2} = 2\sqrt{4-x^2}$.

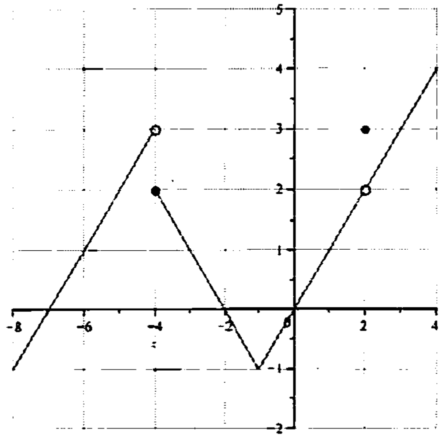
Short Sol'n: Note that $\sin^2(t) + \cos^2(t) = 1$ and $\frac{x}{2} = \cos(t)$ and $\frac{y}{4} = \sin(t)$.

Then $1 = \sin^2(t) + \cos^2(t) = \left(\frac{y}{4}\right)^2 + \left(\frac{x}{2}\right)^2 = \frac{y^2}{16} + \frac{x^2}{4}$

$\frac{y^2}{16} = 1 - \frac{x^2}{4} \implies y^2 = 16\left(1 - \frac{x^2}{4}\right) = 16 - 4x^2$ and so $y = \pm\sqrt{16-4x^2}$

$= \pm 2\sqrt{4-x^2}$.

7. (6 points) Consider the function g graphed below. For what values of x_0, $-8 < x_0 < 4$, does $\lim_{x \to x_0} g(x)$ exist?



$\lim_{x \to x_0} g(x)$ does not exist at $x_0 = -4$

$\lim_{x \to x_0} g(x)$ does exist for all other values of x_0, i.e. $-8 < x_0 < 4$, $x_0 \neq -4$

or $\{-8 < x_0 < -4\} \cup \{-4 < x_0 < 4\}$.

8. (14 points) Sketch the graph of a function f with all of the following properties:

(a) the domain of f is $(0, \infty)$

(b) $f(x) < 0$ if $0 < x < 1$

(c) $f(1) = 1$

(d) $\lim_{x \to 1} f(x) = -2$,

(e) $\lim_{x \to 2^+} f(x) = 1$,

(f) $\lim_{x \to 2^-} f(x) = 2$.

(g) the y-axis is a vertical asymptote for the graph of f