MATH 141

Fundamental Theorem of Calculus (FTC)

Let $f\colon [a,b] \to \mathbb{R}$ be a continuous function.

Let $F\colon [a,b] \to \mathbb{R}$ be an antiderivative of f (recall, this means $F'(x) = f(x)$).

Let $x \in (a,b)$. Then :

$$\int_a^b f(x)\,dx \equiv \int_a^b F'(x)\,dx = F(b) - F(a)$$

and

$$D_x\left[\int_a^x f(t)dt\right] = f(x) \;.$$

Basic Differentiation Rules

If the functions $y = f(x)$ and $y = g(x)$ are differentiable at x and a and b are constants, then:

1. $D_x\left[af(x) + bg(x)\right] = af'(x) + bg'(x)$
2. $D_x\left[f(x) \cdot g(x)\right] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
3. $D_x\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$ provided $g(x) \neq 0$
4. $D_x\left[f(x)\right]^r = r\left[f(x)\right]^{r-1} f'(x)$

If f is differentiable at x and g is differentiable at $f(x)$, then:

5. $D_x\left[g(f(x))\right] = g'(f(x))\, f'(x)$

Exp and Log

DERIVATIVES	$\xrightarrow{\text{FTC}}$	INTEGRALS
$D_x a^x = a^x \ln a$		$\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C$
$D_x \log_a \lvert x\rvert = \dfrac{1}{x \ln a}$		$\displaystyle\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$(\log_a b)(\log_b c) = \log_a c$		$\log_a x = \dfrac{\ln x}{\ln a}$

$\boxed{\text{Trig}}$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \qquad \sin(2x) = 2 \sin x \cos x$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2} \qquad \cos(2x) = \cos^2 x - \sin^2 x$$

DERIVATIVES $\overset{\text{FTC}}{\Longrightarrow}$ INTEGRALS

$$D_x \sin u = \cos u \, \frac{du}{dx} \qquad \int \cos u \, du = \sin u + C$$

$$D_x \tan u = \sec^2 u \, \frac{du}{dx} \qquad \int \sec^2 u \, du = \tan u + C$$

$$D_x \sec u = \sec u \tan u \, \frac{du}{dx} \qquad \int \sec u \tan u \, du = \sec u + C$$

$$D_x \cos u = - \sin u \, \frac{du}{dx} \qquad \int \sin u \, du = - \cos u + C$$

$$D_x \cot u = - \csc^2 u \, \frac{du}{dx} \qquad \int \csc^2 u \, du = - \cot u + C$$

$$D_x \csc u = - \csc u \cot u \, \frac{du}{dx} \qquad \int \csc u \cot u \, du = - \csc u + C$$

MORE INTEGRALS

$$\int \tan u \, du = - \ln |\cos u| + C = \ln |\sec u| + C$$

$$\int \cot u \, du = \ln |\sin u| + C = - \ln |\csc u| + C$$

$$\int \sec u \, du = \ln |\sec u + \tan u| + C = - \ln |\sec u - \tan u| + \widetilde{C}$$

$$\int \csc u \, du = - \ln |\csc u + \cot u| + C = \ln |\csc u - \cot u| + \widetilde{C}$$

INVERSE TRIG FUNCTIONS

$y = \sin\theta$	$\Leftrightarrow$	$\theta = \sin^{-1} y$	where	$-1 \le y \le 1$	and	$\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$
$y = \cos\theta$	$\Leftrightarrow$	$\theta = \cos^{-1} y$	where	$-1 \le y \le 1$	and	$0 \le \theta \le \pi$
$y = \tan\theta$	$\Leftrightarrow$	$\theta = \tan^{-1} y$	where	$y \in \mathbb{R}$	and	$\frac{-\pi}{2} < \theta < \frac{\pi}{2}$
$y = \cot\theta$	$\Leftrightarrow$	$\theta = \cot^{-1} y$	where	$y \in \mathbb{R}$	and	$0 < \theta < \pi$
$y = \sec\theta$	$\Leftrightarrow$	$\theta = \sec^{-1} y$	where	$\lvert y\rvert \ge 1$	and	$0 \le \theta \le \pi\ ,\ \theta \ne \frac{\pi}{2}$
$y = \csc\theta$	$\Leftrightarrow$	$\theta = \csc^{-1} y$	where	$\lvert y\rvert \ge 1$	and	$\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}\ ,\ \theta \ne 0$

DERIVATIVES	$\overset{\text{FTC}}{\Longrightarrow}$	INTEGRALS
$D_x \sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\ \frac{du}{dx}$	$\lvert u\rvert \le 1$	$\int \frac{1}{\sqrt{1-u^2}}\, du = \sin^{-1} u\ +\ C$
$D_x \tan^{-1} u = \frac{1}{1+u^2}\ \frac{du}{dx}$		$\int \frac{1}{1+u^2}\, du = \tan^{-1} u\ +\ C$
$D_x \sec^{-1} u = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\ \frac{du}{dx}$	$\lvert u\rvert \ge 1$	$\int \frac{1}{u\sqrt{u^2-1}}\, du = \sec^{-1} \lvert u\rvert\ +\ C$
$D_x \cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\ \frac{du}{dx}$	$\lvert u\rvert \le 1$	
$D_x \cot^{-1} u = \frac{-1}{1+u^2}\ \frac{du}{dx}$		
$D_x \csc^{-1} u = \frac{-1}{\lvert u\rvert\sqrt{u^2-1}}\ \frac{du}{dx}$	$\lvert u\rvert \ge 1$	