MATH 141 (Section 1 & 2)
Prof. Meade

University of South Carolina
Fall 2004

Exam 2 – Practice
October 6, 2004

Name: ______________________
SS #: ______________________

Instructions:

1. There are a total of 6 problems on 4 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.
5. **This exam is a little longer than what I expect you to be able to complete in a one hour class. But, it is good practice when studying for the exam.**

Problem	Points	Score
1	15	
2	10	
3	35	
4	10	
5	15	
6	15	
Total	100	

Study Hard! Ask Questions!!

1. (15 points) Evaluate the following limits. If a limit does not exist, explain why.

   (a) $\displaystyle\lim_{x\to 0}\frac{2x+\sin x}{x}$

   (b) $\displaystyle\lim_{x\to 0}\frac{\tan(7x)}{\sin(3x)}$

   (c) $\displaystyle\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$

2. (10 points) A function f is said to have a *removable discontinuity* at $x=c$ if $\lim_{x\to c} f(x)$ exists but f is not continuous at $x=c$, either because $f(c)$ is not defined or because $f(c)\neq\lim_{x\to c} f(x)$.

   Let $f(x)=\dfrac{x^2-1}{x-1}$.

   (a) Explain why f has a removable singularity at $x=1$.

   (b) Let $F(x)=f(x)$ for all $x <> 1$. How should $F(1)$ be defined in order to make g continuous at $x=1$?

3. (35 points) Find the requested derivative in each part.

   (a) Find $f'(x)$ when $f(x) = x^3 - 3x^2 + 4\sqrt{x}$

   (b) Find $\frac{dy}{dx}$ when $y = x\cos(x^2)$

   (c) Find y' when $y = \sqrt{t^2 + 4t + 3}$

   (d) Find $r'(\theta)$ when $r(\theta) = \dfrac{\sin\theta}{\theta}$

   (e) Find $\frac{d^2x}{dt^2}$ when $x = \tan(2t)$

4. (10 points) Find all values of x in the interval $[-2\pi, 2\pi]$ where the graph of $y = x + \cos x$ has a horizontal tangent line.

5. (15 points)

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	$\pi/3$	5	$\pi/2$	-2
3	0	-4	-3	3
5	$\pi/4$	2	$\pi/6$	2

Use the table of values shown above to find

(a) $F'(2)$ when $F(x) = f(g(x))$

(b) $G'(2)$ when $G(x) = (g(x))^2$

(c) $H'(2)$ when $H(x) = \dfrac{g(x)}{\cos(f(x))}$

6. (15 points) The equation $\frac{1}{x} + \frac{1}{y} = y$ implicitly defines y as a function of x.

(a) Use implicit differentiation to find $\frac{dy}{dx}$.

(b) Find all points on the graph of this function with $y = 2$.

(c) For each point found in (b), find the equation of the tangent line to the curve.