MATH 141 (Section 1 & 2)
Prof. Meade

University of South Carolina
Fall 2004

Exam 1
September 15, 2004

Name: Key
SS #: ____________

Instructions:

1. There are a total of 7 problems on 6 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	12	
2	12	
3	16	
4	12	
5	12	
6	21	
7	15	
Total	100	

1. (12 points) [2 points for each blank] Short Answer. *Fill in the blank with the word, equation, or short phrase that best completes each statement.*

   $2x+3y=5$
   $3y=5-2x$
   $y=\frac{5}{3}-\frac{2}{3}x$

   (a) The equation $2x+3y=5$ represents a line with slope $\underline{-2/3}$.

   (b) $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^-} f(x) = \underline{L}$ and $\lim_{x\to a^+} f(x) = \underline{L}$.

   (c) The limit property for quotients states that

   $$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \textit{ except} \text{ when } \underline{\lim_{x\to a} g(x) = 0}.$$

   (d) The line $y=3$ is a horizontal asymptote of the graph of $y=f(x)$, then either

   $\underline{\lim_{x\to+\infty} f(x)=3}$ or $\underline{\lim_{x\to-\infty} f(x)=3}$.

2. (12 points) [4 points each] Find the natural domain of each of the following functions.

   (a) $f(x) = \dfrac{x-2}{x^2-4}$

   $x^2-4=0$ when $x=\pm 2$

   domain is all x except $x=\pm 2$

   or, $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

   (b) $g(\theta) = \sin(\theta)$

   domain is all real numbers

   (c) $h(t) = \dfrac{\sqrt{x-2}}{\sqrt[3]{x}-2}$

   $\sqrt{x-2}$ has domain $x\geq 2$

   $\sqrt[3]{x}$ has domain $(-\infty,+\infty)$

   but you can't divide by zero and

   $\sqrt[3]{x}-2=0$ when $x^{1/3}=2$
   $x=8$.

   So, the domain of h is $x\geq 2$ except $x=8$

   or, $[2,8)\cup(8,\infty)$.

3. (16 points) Let $f(x) = 2|x| - 2x + 3$.

(a) [10 points] Express $f(x)$ in piecewise form without using absolute values.

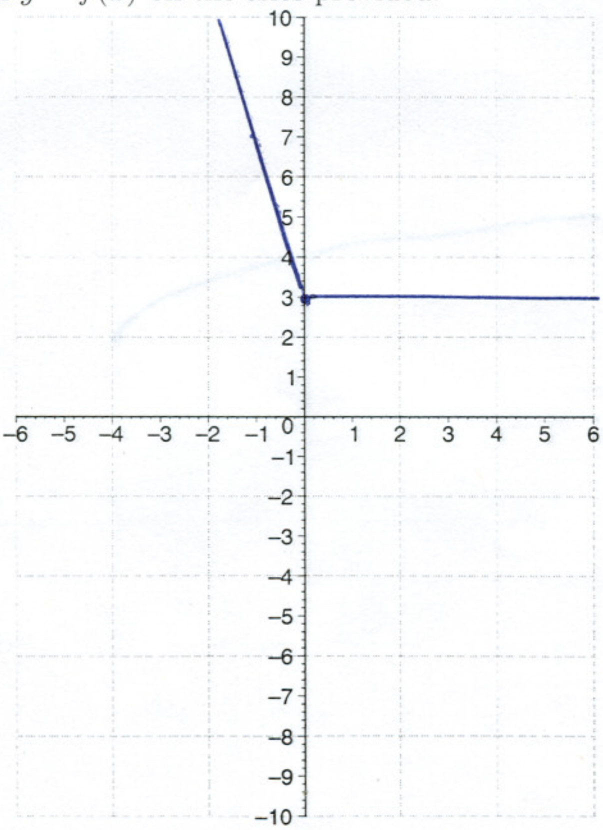
$$f(x) = 2|x| - 2x + 3$$

$$= 2\begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} - 2x + 3$$

$$= \begin{cases} 2x - 2x + 3 & \text{if } x \geq 0 \\ -2x - 2x + 3 & \text{if } x < 0 \end{cases}$$

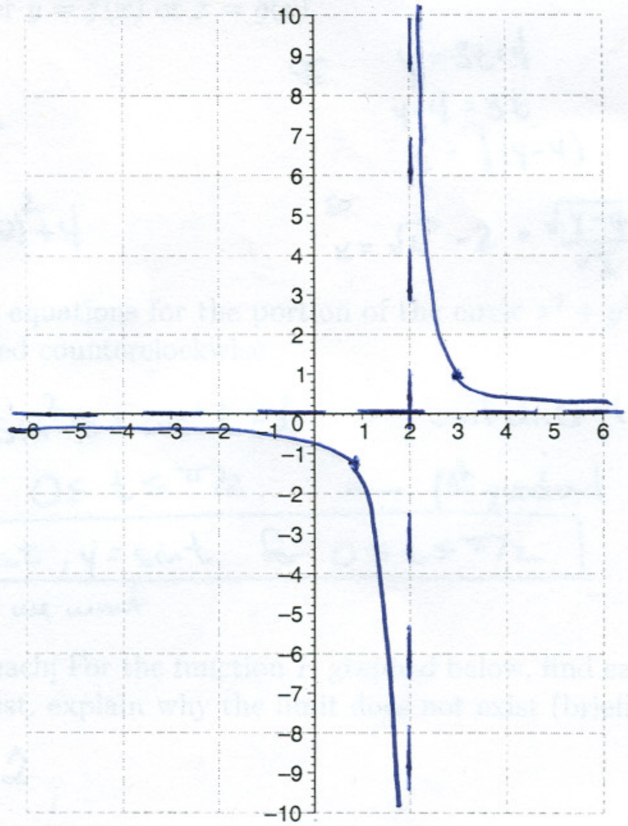
$$= \begin{cases} 3 & \text{if } x \geq 0 \\ -4x + 3 & \text{if } x < 0 \end{cases}$$

(b) [6 points] Graph $y = f(x)$ on the axes provided.



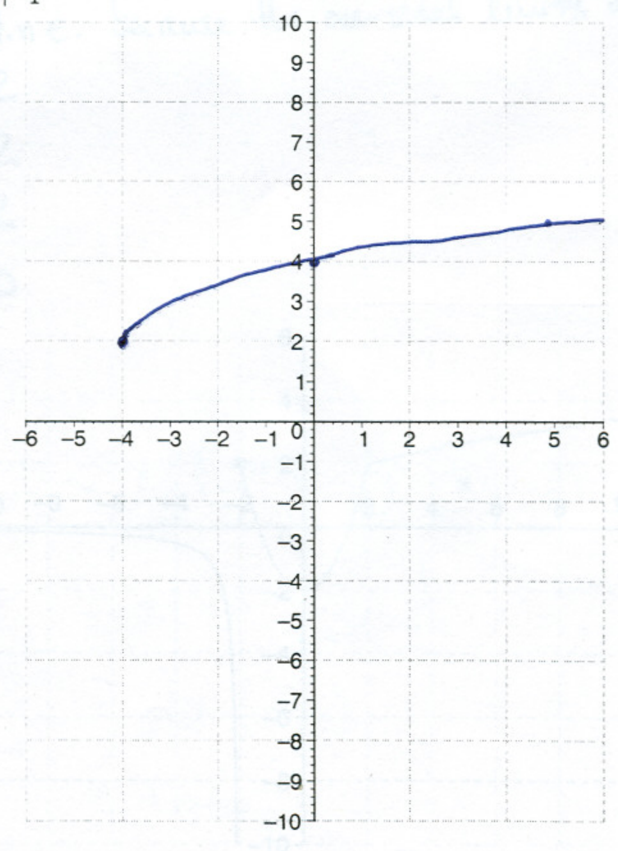
4. (12 points) [6 points each] Sketch the graph of the following functions on the axes provided.

(a) $f(x) = \dfrac{1}{x-2}$



the graph of $y = \frac{1}{x}$ shifted 2 to the right.

(b) $f(x) = 2 + \sqrt{x+4}$



$f(0) = 2 + \sqrt{4} = 4$

$f(5) = 2 + \sqrt{9} = 5$

the graph of $y = \sqrt{x}$ shifted 4 to the left and up 2.

5. (12 points) [6 points each]

(a) Consider the parametric curve with $x = \sqrt{t} - 2$ and $y = 3t + 4$. Express this curve in the form of either $y = f(x)$ or $x = g(y)$.

$x = \sqrt{t} - 2$
$x + 2 = \sqrt{t}$
$t = (x+2)^2$

so

$y = 3t + 4 = 3(x+2)^2 + 4$

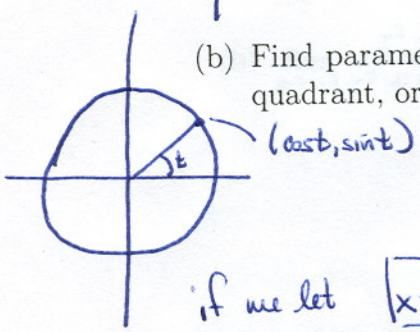
or

$y = 3t + 4$
$y - 4 = 3t$
$t = \frac{1}{3}(y-4)$

so

$x = \sqrt{t} - 2 = \frac{\sqrt{y-4}}{\sqrt{3}} - 2.$

(b) Find parametric equations for the portion of the circle $x^2 + y^2 = 1$ that lies in the first quadrant, oriented counterclockwise.



$\sin^2 t + \cos^2 t = 1$ ← unit circle (t is the angle w/ the x-axis)

$0 \le t \le \pi/2.$ ← 1st quadrant

if we let $\boxed{x = \cos t,\ y = \sin t \text{ for } 0 \le t \le \pi/2}$

we will have what we want

6. (21 points) [3 points each] For the function F graphed below, find each of the following limits. If a limit does not exist, explain why the limit does not exist (briefly).

(a) $\lim_{x \to -2^+} F(x) =$ 2

(b) $\lim_{x \to -2^-} F(x) =$ $-\infty$

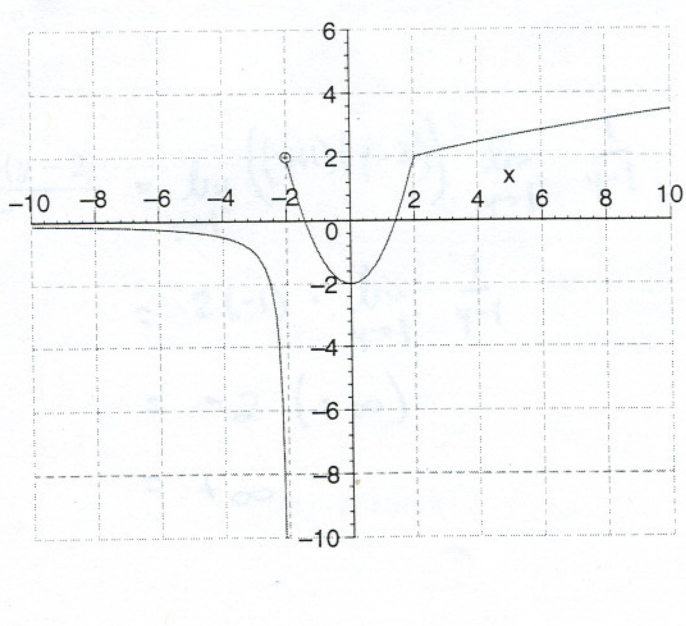
(c) $\lim_{x \to -2} F(x) =$ d.n.e. because the one-sided limits are not equal.

(d) $\lim_{x \to 2^-} F(x) =$ 2

(e) $\lim_{x \to 2^+} F(x) =$ 2

(f) $\lim_{x \to 2} F(x) =$ 2

(g) $\lim_{x \to -\infty} F(x) =$ 0



7. (15 points) [3 points each] Find the following limits.

(a) $\lim_{y \to 6^+} \frac{y+6}{y^2+36}$ $= \frac{6+6}{36+36} = \frac{12}{72} = \frac{1}{6}$

(b) $\lim_{x \to -3} \frac{x^2-9}{x^2+x-6}$ $= \lim_{x \to -3} \frac{(x-3)(x+3)}{(x-2)(x+3)}$

$= \lim_{x \to -3} \frac{x-3}{x-2}$

$= \frac{-6}{-5} = 6/5 = 1.2$

(c) $\lim_{x \to \infty} \frac{x^2-9}{x^2+x-6}$ $= \lim_{x \to \infty} \frac{x^2(1 - 9/x^2)}{x^2(1 + \frac{1}{x} - \frac{6}{x^2})}$

$= \lim_{x \to \infty} \frac{1 - 9/x^2}{1 + 1/x - 6/x^2} = \frac{1}{1} = 1.$

(d) $\lim_{y \to 2^-} \frac{(y+1)(y-2)}{y-1}$ $= \frac{3(0)}{1} = 0$

(e) $\lim_{y \to 1^-} \frac{(y+1)(y-2)}{y-1}$ $= \lim_{y \to 1^-} \left((y+1)(y-2)\right) \cdot \lim_{y \to 1^-} \frac{1}{y-1}$

$= 2(-1) \cdot \lim_{y \to 1^-} \frac{1}{y-1}$

$= -2(-\infty)$

$= +\infty$