

# Geometry in the 21<sup>st</sup> Century Dynamic and Symbolic

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# Outline

- Euclidean Geometry
- Conic Sections
- Shrinking Circles

# Euclidean Geometry

## ■ Napoleon's Theorem

- ◆ Given any  $\triangle ABC$
- ◆ Construct an equilateral triangle on each side of  $\triangle ABC$
- ◆ Construct  $\triangle GIK$  joining the incenters of the three equilateral triangles

◆ What can be said about  $\triangle GIK$ ?



# Conic Sections

- Ellipse
- Hyperbola
- Parabola

# Ellipse

## ■ Definition

An **ellipse** is a curve that is the locus of all points in the plane the *sum* of whose distances from two fixed points (the foci) is a given positive constant.



# Hyperbola

## ■ Definition

A **hyperbola** is a curve defined as the locus of all points in the plane the *difference* of whose distances from two fixed points (the foci) a given positive constant.



# Parabola

## ■ Definition

A **parabola** is the set of all points in the plane equidistant from a given line (the conic section directrix) and a given point not on the line (the focus).



# Shrinking Circles

## ■ Construction

- ◆  $C^*$  – unit circle centered at  $(1,0)$
- ◆  $C_r$  – circle with radius  $r$  centered at origin
- ◆  $P$  – a  $(0, r)$
- ◆  $Q$  – intersection of  $C^*$  and  $C_r$  in first quadrant
- ◆  $R$  – intersection of the line  $PQ$  and the  $x$ -axis



# Shrinking Circles

## ■ Question

What happens to  $R$  as  $C_r$  shrinks  
i.e., in the limit as  $r$  decreases towards 0?



# Generalization

- Suppose  $C^*$  is replaced with an ellipse that passes through the origin:
  - ◆ What happens to  $R$ ? as  $C_r$  shrinks?
  - ◆ Does it matter if an axis of the ellipse is on the  $x$ -axis.
- What if  $C^*$  is replaced with a general curve that passes through the origin?



# Generalization

## ■ 3-D Construction

- ◆  $S^*$  – unit sphere centered at  $(1,0,0)$
- ◆  $S_r$  – sphere with radius  $r$  centered at origin
- ◆  $P$  – a  $(0, 0, r)$
- ◆  $C$  – intersection (curve) of  $S^*$  and  $S_r$
- ◆  $R$  – intersection (curve) of cone with vertex  $P$  and cross-section  $C$  with the  $xy$ -plane



# Conclusions

## ■ Geometry

- ◆ is more than pencil and paper constructions
- ◆ is more than two-column proofs
- ◆ can be dynamic
- ◆ is relevant to calculus,

and beyond

# Relevant Software

- Geometry Expressions (beta)  
<http://www.geometryexpressions.com/>
- Geometer's Sketchpad  
<http://www.keypress.com/sketchpad/>
- Cabri3D  
<http://www.cabri.com/>
- Maple  
<http://www.maplesoft.com/>

