Graphical Analysis in Polar Coordinates

Douglas B. Meade

Department of Mathematics

Overview

One of the most challenging aspects to polar coordinates is being able to visualize the graph of a polar function, $r = f(\theta)$. An animation showing exactly how the curve is traced out as the angle moves through its domain is even more useful than a static graph of the function.

The simplest polar plots can be created with the `plot` command — with one additional argument. To create an animation in polar coordinates it is easier to work with a *parametric form* of the equation. (Parametric curves will be discussed in more detail in Calculus III.)

Preparation

- Know the basic conversions between rectangular and polar coordinates:

$$\begin{aligned} r &= \sqrt{x^2+y^2} & \qquad x &= r\cos(\theta) \\ \tan\theta &= \tfrac{y}{x} & \qquad y &= r\sin(\theta) \end{aligned}$$

- Remember that all angles need to be specified in radians.
- Be prepared to create some surprising plots that would be almost impossible to create in rectangular coordinates.

Maple Essentials

- The *PolarCurveID* and *Basic14Polar* maplets are available at USC from the URLs:

```
http://www.math.sc.edu/~meade/142L-S05/maplets/CalcUSC/PolarCurveID.maplet
http://www.math.sc.edu/~meade/142L-S05/maplets/CalcUSC/Basic14Polar.maplet
```

- New Maple commands introduced in this lab include:

Command	Description
`arctan( y, x )`	two-argument version of the inverse tangent this is essentially equivalent to `arctan(y/x)` except that the signs of `x` and `y` are used to extend the range from $(-\frac{\pi}{2}, \frac{\pi}{2})$ to $(-\pi, \pi)$; this modification makes the two-argument arctan ideal for converting from rectangular to polar coordinates
`plot( ..., coords=polar);`	plot a function in polar coordinates the most common usage is: > `R := 2*cos(4*t)` > `plot( R, theta=0..2*Pi, coords=polar );`
`animatecurve`	animated sketch of a curve e.g., the limaçon $r = 1 + 3\sin(\theta)$ could be animated as follows: > `R := 1 + 3*sin( t );` > `animatecurve( [ R, t, t=0..2*Pi ], coords=polar;` **Note:** *Execute* `with( plots ):` *before using* `animatecurve`.
`unassign`	remove assignments from a Maple name to prevent the name from evaluating to its value, it is necessary to enclose each name in single quotes, e.g., > `unassign( 'x', 'y', 'r' );`

Assignment

- Mastery Quiz 11 will be distributed during this lab. This quiz consists of two parts. The first part (worth two points) is a commitment to complete an end-of-course survey. The second part (worth 1 point) is a question that will be answered during this hour. Your TA has instructions for turning in the survey.
- Have a great summer!
  (But, do not forget all of your calculus, or all of the Maple, you have learned this year.)

Activities

1. Convert the following points to polar coordinates: (2,0), (3,3), (0,2), (-2,3), (-2,-5), (0,-3), $(1,-\sqrt{3})$. **Note:** *Compare the angles obtained with* `arctan(y/x)` *and* `arctan(y,x)`*.*
2. Create plots of the unit circle, $x^2 + y^2 = 1$, in both rectangular and polar coordinates.
   Note: *In which coordinate system is it easier to plot the unit circle?*
3. Plot each of the following curves in polar coordinates.

(i)	$r = 2 + \sin(\theta)$	(ii)	$r = \cos(4\theta)$	(iii)	$r = 3(1 - \cos(\theta))$
(iv)	$r = \sin\left(\frac{\theta}{5}\right)$	(v)	$r = \sin(\theta) + \cos\left(\frac{\theta}{3}\right)$	(vi)	$r = 2 + \sin\left(\frac{5\theta}{3}\right)$
(vii)	$r = \ln(\theta)$	(viii)	$r = \frac{\theta}{2}$	(ix)	$\theta = \frac{\pi}{3}$
(x)	$r = 1 + (\cos(\theta))^3$	(xi)	$r = (\cos(\theta))^2$	(xii)	$r^2 = \cos(2\theta)$

   Note: *If a curve is periodic, determine the period.*
4. Animate the sketching of each curve in Activity 2.
   Hint: *A polar function $r = f(\theta)$ can be written in parametric form as $r = f(t)$, $\theta = t$.*
   Note: *Optional arguments to the* `animatecurve` *command include:*
   - `frames=`*num creates an animation with num frames; the default number of frames is 16.*
   - `numpoints=`*num instructs Maple to use num points in each frame of an animation; the default number of points is 50.*
5. The polar function $r = e^{\cos(\theta)} - 2\cos(4\theta) + \left(\sin\left(\frac{\theta}{4}\right)\right)^3$ is called the "butterfly curve".
   (a) Find a parameter interval that traces this curve exactly once.
   (b) Plot or animate the curve.
6. The area inside the polar curve $r = 5\cos(3\theta)$ can be found by following these steps:
   (a) Plot the curve.
   (b) Find an appropriate parameter interval that traces the curve exactly once.
   Hint: *An animation can help. Increase the number of frames in the animation as needed.*
   (c) Set up a definite integral (in polar coordinates) for the area of one leaf.
   (d) Evaluate the definite integral and find the total area inside all three leaves.
   (e) Check that this answer is consistent with your plot.
7. Let R be the region outside $r = 3\sqrt{3}\cos(\theta)$ and inside $r = 3\sin(\theta)$. The area of R can be found as follows:
   (a) Divide the region into two pieces; R_1 whose area can be found by a simple formula and R_2 that is more complicated.
   (b) Find the intersection between the two curves in the first quadrant.
   (c) Find an appropriate parameter interval for region R_2.
   (d) Set up a definite integral (in polar coordinates) for the area of R_2.
   (e) Evaluate the definite integral and find the total area of R.
   (f) Check that this answer is consistent with your plot.