# Tangent Lines and Differentiation Rules

**Objective**   This lab provides practice applying the differentiation rules and using derivatives to answer questions about tangent lines and rates of change.

**Background**   Differentiation is very methodical. You apply the basic differentiation rules in the correct order and you eventually come to an expression for the derivative of the original expression. The `Differentiation` maplet is a "differentiation calculator" designed to help you learn the correct order in which to apply the differentiation rules for any particular problem. A complete list of maplets for Calculus I is maintained on the WWW at

> http://www.math.sc.edu/~meade/141L-F03/maplets/.

As you become comfortable with the computation of derivatives and begin to need to use derivatives to answer other questions, Maple's `diff` and `Diff` commands should be used to return the derivative of an *expression*. The `D` command returns the derivative function of a given *function*.

Other new commands encountered in this lab include `rhs`, to extract the right-hand side of an equation, `isolate`, to solve an equation for a quantity (that could be more complicated than a name), `unapply`, to create a Maple function from a Maple expression, and, from the `Student[Calculus1]` package, the `Tangent` command for quickly returning the equation of a tangent line at a point on the graph of a function. Lastly, square brackets can be used to refer to a specific element in a list, e.g., `L[2]`.

**Discussion**   The `Differentiation` maplet can be described as a calculator, or tutor, specifically for the computation of derivatives. A maplet can be run on your local computer if Maple 8 is installed. Otherwise, MapleNet is used to run the maplet remotely. All that is needed to run a maplet over the web is a fairly recent version of Java.

Example 1: The `Differentiation` maplet
- From your browser, launch the `Differentiation` maplet
- Enter a function, say `x^2/(x^2+1)`
- Enter the name of the independent variable, typically `x`
- Press the `Start` button in the maplet window
- The first step is to apply the *Quotient Rule*, i.e., press `Quotient`
- To compute $\frac{d}{dx}x^2$, apply the *Power Rule*.
- To compute $\frac{d}{dx}(x^2+1)$, apply the *Sum Rule*.
- To conclude, apply the *Constant Rule* and the *Power Rule* (again).
- The final step is to simplify Maple's answer by hand.

Enter, and execute, the following Maple commands in a Maple worksheet.

Example 2: Derivatives of Expressions

```
> restart;                              # clear Maple's memory
> F := x^4 - 2*x^3 - 7*x^2 + 20*x - 12; # a quartic expression
> Fp := diff( F, x );                   # first derivative
> plot( [ F, Fp ], x = -3 ..  3,        # plot F and F'
>       color=[red,blue],
>       legend=["y=F","y=F'"] );
> q1 := Fp = 0;                         # eqn for pts w/horiz tangent
> solve( q1, {x} ) ;                    # three solutions
> Fppp := diff( F, x,x,x );             # third derivative
> eval( Fppp, x=4 );                    # evaluate F'''(4)
```

Example 3: Derivatives of Functions

```
> f := unapply( F, x );                          # convert expression to function
> Df := D( f );                                   # first derivative fn
> plot( [ f(x), Df(x) ], x = -3 ..  3,            # plot f and f'
>       color=[red,blue],
>       legend=["y=f(x)","y=f'(x)"] );
> q2 := Df(x) = 0;                                # eqn for pts w/horiz tangent
> solve( q2, {x} ) ;                              # three solutions
> D3f := D(D(D( f )));                            # third derivative fn
> D3f( 4 );                                       # evaluate f'''(4)
```

Example 4: Tangent Lines

```
> pt := [ -2, f(-2) ];                            # pt on graph of y = f(x)
> m := Df( pt[1] );                               # slope = deriv at point
> q3 := m = ( y - pt[2] ) / ( x - pt[1] );        # slope = rise / run
> Ltan := isolate( q3, y );                       # another way to solve for y
> plot( [ f(x), rhs(Ltan) ], x=-4..4,             # plot of fn and tangent line
>       legend=["Function","Tangent line");
> with( Student[Calculus1] );                     # load package
>  Tangent( f(x), x=-2 );                          # tangent line
```

**Notes**

(1) If you are uncertain about the next rule to apply in the `Differentiation` maplet, click the `Obtain a Hint` button.

(2) It can be very tedious to repeatedly apply some of the simpler differentiation rules, e.g., `Constant` and `Constant Multiple`. To have Maple automatically apply a rule, place a check mark next to each rule in the `Understood Rules` menu that you want to be applied automatically.

(3) The `TangentAndSecant`, `TangentLinePlot`, and `DerivativePlot` maplets are also relevant at this point in the course. The first maplet can show an animation of the convergence of secant lines to the tangent line to the graph of a function at a point. The second maplet displays the equation and graph of the tangent line to the graph at a point. The third maplet is used to create a graph of the function and one or more derivatives of the function.

**Questions**

(1)  (a) Assume $f_1$, $f_2$, $f_3$, and $f_4$ are differentiable functions. Express

$$\frac{d}{dx}\left( f_1(x)f_2(x)f_3(x)f_4(x) \right).$$

as the sum of four terms. How many times is the product rule used to obtain this answer?

(b) How many times would the product rule be used to find the derivative of the product of $n$ differentiable functions?

(2)  (a) Find the $x$-coordinate for each point on the graph of $y = \sqrt{4x - x^2}$ whose tangent line passes through the point $(3, 6)$.

(b) For what values of $y_0$ are there two, one, and no tangent lines to the graph of $y = \sqrt{4x - x^2}$ that pass through the point $(3, y_0)$?

(3) Let $g(t) = \cos(t^3 - 3t + \sin(2t))$. Find $g$ and its first 5 derivatives. Classify each of these six functions as even, odd, or neither. Based on these results, for what values of $n$ is the $n^{\text{th}}$ derivative of $g$ an even function?