Linear Motion Problems

Objective This lab assignment asks you for some information about objects that move back and forth along a line – a linear motion problem.

Background You will need to understand the relationships between position, velocity, and acceleration. You also need to be able to recognize the global extrema of a function on a closed interval.

The previous labs have introduced almost all of the Maple needed to complete this lab. Several of the questions call for the solution of a polynomial equation. While Maple is pretty good at solving polynomial equations, you should always verify that all solutions have been found. A graph is often helpful at this stage but there are some instances where this is not suitable. In these instances you should remember the Fundamental Theorem of Algebra: A polynomial $p(x)$ of degree n has exactly n real and complex roots. Moreover, when the coefficients of the polynomial are real-valued, any complex roots occur in complex conjugate pairs. That is, if $z = a + ib$ is a solution to $p(z) = 0$, then so is $\bar{z} = a - ib$.

The `solve` command can be used to solve inequalities as well as equalities. Many of the results returned by `solve` contain one or more occurrences of `RootOf`. You already know that the `allvalues` command can be used to request the explicit form for these roots. If a floating-point approximation to the root is acceptable, then there are a couple of options. First, `evalf` can be applied to the output from `allvalues`. Alternatively, if the original equation submitted to `solve` contains at least one floating point number then Maple will return its answer as floating point numbers. This completely avoids the `RootOf`s and the need for `allvalues`.

Lastly, the `select` and `remove` commands can be used to retain or reject items of a set or list based on specified criteria. In particular, the command `remove(` f `,` e `);` assumes f is a Boolean-valued function (i.e., a function that returns a value of TRUE or FALSE) and e is a Maple expression with several operands (e.g., the elements of a list or set). This command applies f to every operand of e and *removes* all operands that return TRUE. (`select(` f `,` e `);` does the opposite – all operands of e that yield TRUE from f are *selected*.)

Discussion Enter, and execute, the following Maple commands in a Maple worksheet.

Example 1: Working with the Results from `solve`

```
> restart;                                   # clear Maple's memory
> F := x -> x^6 - x^4 + x^3 + x;             # define polynomial
> plot( F(x), x=-2..1, y=-5..10 );           # plot function
> eq1 := F(x) = 0;                           # equation (exact)
> s1 := solve( eq1, {x} );                   # solutions – as expr seq
> s2 := allvalues( [s1] );                   # not very useful
> s3 := evalf( [s1] );                       # 6 real and complex roots
> s4 := remove( has, s3, I );                # remove complex roots
> eq2 := F(x) = 0.;                          # equation (floating point)
> s5 := solve( eq2, {x} );                   # 6 real & complex roots - approx
> s6 := remove( has, [s5], I );              # remove complex roots
```

Example 2: Solving Inequalities

```
> ineq := F(x) > 10;                # create inequality
> q1 := solve( ineq, x );           # solution w/RootOf
> q2 := evalf( [q1] );              # floating point approx
```

Questions

(1) An object moves along the horizontal coordinate line according to the formula

$$s = t^3 - 20t^2 + 60t - 30 + \frac{16}{t}$$

where s is the directed distance from the origin in feet and $t > 0$ is time in seconds.

- Find the velocity (as a function of time).
- Find the speed (as a function of time).
- Find the acceleration (as a function of time).
- Find all times when the object is moving to the right.
- Find all times when the object's acceleration is negative.
- Create a well-labeled graph showing the position, velocity, speed, and acceleration on a window that confirms all of the other results for this problem.
- On the interval $1 \leq t \leq 15$, what is the greatest distance between the object and the origin? When does this occur?
- When does the object attain its highest velocity on the interval $1 \leq t \leq 15$? What is this velocity?

(2) Two particles move along a coordinate line. Both objects begin at the origin at time $t = 0$. After t seconds (assume $t > 0$) their directed distances from the origin, in feet, are given by

$$s_1 = \frac{1}{400}(3t^5 - 8t^4 + 50t)$$

and

$$s_2 = 8t^2 - 12t - t^3,$$

respectively.

- When do the objects have the same position?
- When do the objects have the same velocity?
- When do the objects have the same speed?
- In general, which is larger: the number of times two objects have the same velocity or the number of times two objects have the same speed? (Explain.)