Antiderivatives

Objective In this lab you will develop your understanding and manipulative abilities with antiderivatives.

Background An antiderivative of a function f is any function F which satisfies the condition

$$F'(x) = f(x).$$

Recall that antiderivatives are not unique. If F is an antiderivative of f, then the general antiderivative of f is $F(x) + C$

The basic Maple command for antiderivatives is `int`. The syntax for $\int f(x)\,dx$ is `int( f(x), x );`. Note, however, that Maple reports *an* antiderivative. If you need the general antiderivative it will be necessary for you to include an appropriate arbitrary constant.

The `Antiderivative` maplet is an interface to the visual relationship between a function and its antiderivatives. The `Integration` maplet is a calculator-like interface with buttons corresponding to each of the primary rules for evaluating integrals.

Discussion Enter, and execute, the following Maple commands in a Maple worksheet.

Example 1: The `int` Command

```
> restart;                                  # clear Maple's memory
> f := x -> x^2 - 3 + 8*sin(x);             # define integrand
> F := int( f(x), x );                      # an antiderivative
> diff( F, x );                             # verify that F'(x) = f(x)
> G := int( f(x), x ) + C;                  # general antiderivative
> diff( G, x );                             # verify that F'(x) = f(x)
```

Example 2: The `Antiderivative` Maplet

- launch the `Antiderivative` maplet
- in the **Function** box, enter `x^2 - 3 + 8*sin(x)`
- in the **a** = and **b** = boxes, enter `-2*Pi` and `2*Pi`, respectively
- press **Plot**
- in the **Value** box, enter `[ 0, 0 ]`; press **Plot**
- place a check in the *Show class of antiderivatives* checkbox; press **Plot**

Observe that the local extrema of each antiderivative (in blue or green) occur at the points where the function (in red) is zero.

Example 3: The `Integration` Maplet

- launch the `Integration` maplet
- in the **Function** box, enter `x^2 - 3 + 8*sin(x)`
- in the **Variable** box, enter `x`
- to start the evaluation of this antiderivative, press **Start**
- to apply the *Sum Rule* (twice), press **Sum** (once)
- to evaluate the first integral using the *Power Rule*, press **Power**
- to evaluate the second integral using the *Constant Multiple Rule*, press **Constant Multiple**
- in the **Function Rules** area of the interface, press **Select a Function**, click on **sin**, press **Apply**

Notes

(1) To create a Maple function corresponding to an antiderivative, use the `unapply` command. For example,
`> F := unapply( int( f(x), x ), x );`

(2) The plots displayed in the `Antiderivative` maplet are created with the `AntiderivativePlot` command, from the `Student[Calculus1]` package. The basic syntax is
`> AntiderivativePlot( f(x), x=a..b );`.
To obtain the antiderivative that passes through a specific point (x_0, y_0), include the optional argument `value=[`x_0`,`y_0`]`. To see a family of antiderivatives, include the optional argument `showclass=true`.

(3) To perform a *Generalized Power Rule* with the function $g(x)$, enter `u=g(x)` in the large box in the **Integration Rules with Arguments** region and press the **Change** button. Then, at the end of the problem, press **Revert** to return to the original independent variable.

(4) In the last step for the example with the `Integration` maplet, it is also possible to type the name of the function, e.g., `sin`, in the box instead of working through the **Select a Function** menu.

(5) The **Understood Rules** menu in the **Integration** maplet can be used to identify rules to be applied automatically whenever possible.

(6) The `Student[Calculus1]` package contains commands that correspond to many of the buttons on the `Integration` maplet. There is no reason to explain these here as there is no reason for you to use these commands.

Questions

(1) Use the `Integration` maplet to evaluate $\int \sin^2 x \, dx$. Use the **All Steps** button to obtain a full listing of the steps in the evaluation of this indefinite integral. Summarize this evaluation in your lab report.
Note: If you have trouble with the formatting of mathematical expressions, use Maple notation for integrals. For example, write $\int \sin^2 x \, dx$ as `int( sin(x)^2, x )`.

(2) Use the `Integration` maplet to evaluate each of the following indefinite integrals.
Note: It is not necessary to show all steps, just report the answer.

(a) $\displaystyle\int x\sqrt{x+1}\, dx$

(b) $\displaystyle\int x\sqrt{x^2+1}\, dx$

(c) $\displaystyle\int x^2 \sin x^3 + 1 \, dx$

(d) $\displaystyle\int \sin^3((x^2+1)^4)\cos((x^2+1)^4)(x^2+1)^3 x \, dx$

Hint: For (d), do not use the **All Steps** button. Think! (See also, Note (3).)

(3) Let

$$F(x) = (\sin x + \cos x)^4 \qquad \text{and} \qquad G(x) = 2\sin(2x) - 4\cos^4 x + 4\cos^2 x + 5.$$

Show that F and G are antiderivatives of the same function. Explain why two functions that appear so different can be antiderivatives of the same function. Find the function f with $F'(x) = f(x)$ and $G'(x) = f(x)$.