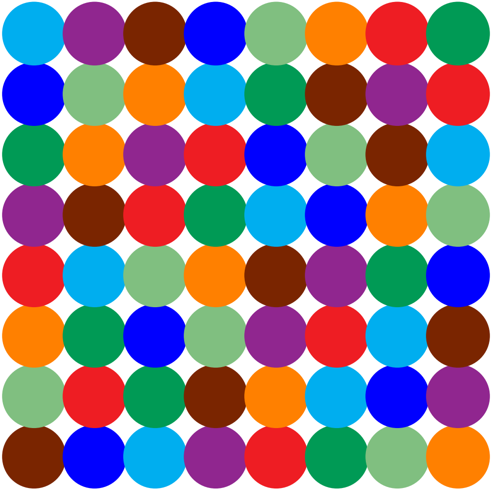

Walter at Asilomar, 1987

Some Varieties are Finitely Based

George F. McNulty

Department of Mathematics
University of South Carolina

WalterFest 2004
Boulder, Colorado

Outline

Outline

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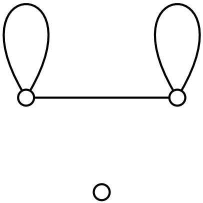
A Flat Graph Algebra **D**



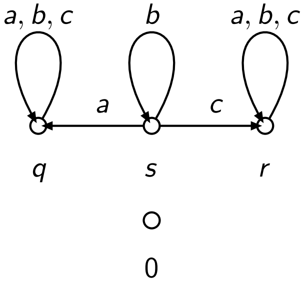
0

Characteristics

A system $\langle V \cup \{0\}, \cdot, \wedge \rangle$ where

- $\langle V, E \rangle$ is a graph with vertex set V and edge set E.
- $0 \notin V$.
- $\cdot$ and $\wedge$ are two-place operations on $V \cup \{0\}$.
- $\cdot$ is determined by adjacency in $\langle V, E \rangle$.
- $\wedge$ is a height 1 semilattice operation with least element 0.

An Automatic Algebra **L**



Characteristics

A system $\langle Q \cup \Sigma \cup \{0\}, \cdot\rangle$ where

- Q, Σ, and $\{0\}$ are disjoint sets.
- $\cdot$ is two-place operation on $Q \cup \Sigma \cup \{0\}$.
- $\cdot$ is determined by a partial automaton with alphabet Σ and state set Q.

Outline

Some Things We Know

Theorem (Cayley, 1854)

The variety of groups is finitely based.

Theorem (Lyndon, 1951)

Every two-element algebra with finitely many fundamental operations is finitely based.

Theorem (Lyndon, 1954)

That seven-element automatic algebra **L** *is* **not** *finitely based.*

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What We Didn't Know

Tarski's Finite Basis Problem (1955-65)

Is there an algorithm which determines whether a finite algebra with finitely many fundamental operations is finitely based?

What We Didn't Know

McKenzie's Resolution of Tarski's Finite Basis Problem (1996)

There is **no** algorithm which determines whether a finite algebra with finitely many fundamental operations is finitely based**!**

Some More Things We Know

Theorem (Oates and Powell, 1964)

Every finite group is finitely based.

Theorem (McKenzie, 1970)

Every finite lattice with finitely many operators is finitely based.

Theorem (Kruse and L'vov, 1973)

Every finite ring is finitely based.

Theorem (Perkins, 1966)

A certain semigroup of six 2×2 matrices under matrix multiplication is **not** *finitely based.*

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Then We Found Out

Theorem (Baker, 1977)

Let $\mathcal{V}$ be a variety of finite signature.
If

- *$\mathcal{V}$ is congruence* **distributive**.
- *$\mathcal{V}$ has a finite residual bound.*

then $\mathcal{V}$ is finitely based.

Then We Found Out

Theorem (McKenzie, 1987)

Let $\mathcal{V}$ be a variety of finite signature.
If

- *$\mathcal{V}$ is congruence* **modular**.
- *$\mathcal{V}$ has a finite residual bound.*

then $\mathcal{V}$ is finitely based.

Then We Found Out

Theorem (Willard, 2000)

Let $\mathcal{V}$ be a variety of finite signature.
If

- *$\mathcal{V}$ is congruence* **meet semidistributive**.
- *$\mathcal{V}$ has a finite residual bound.*

then $\mathcal{V}$ is finitely based.

Anyone's Guess

Park's Conjecture/ Jónsson's Speculation 1974

Let $\mathcal{V}$ be a variety of finite signature.
If

- ....
- $\mathcal{V}$ has a finite residual bound.

then $\mathcal{V}$ is finitely based.

Good, But . . .

The Group $\mathbf{Q}$ of Quaternions

The variety generated by $\mathbf{Q}$

- Is congruence modular.
- Has **no** residual bound.

Nevertheless it is finitely based.

Good, But . . .

The Flat Graph Algebra **D**

The variety generated by **D**

- Is congruence meet semidistributive.
- Has **no** residual bound.

Nevertheless it is finitely based.

Bjarni Gets Beyond a Residual Bound

Theorem (Jónsson 1979)

Let $\mathcal{V}$ be a variety of finite signature.
If

- *$\mathcal{V}$ is congruence distributive.*
- *$\mathcal{V}_{\mathrm{fsi}}$ is finitely axiomatizable.*

then $\mathcal{V}$ is finitely based.

Outline

Translations

Definition

Let $\mathbf{A}$ be an algebra. A function $\lambda : A \to A$ is a

Basic translation provided λ arises from a basic operation of $\mathbf{A}$ by evaluating all but one of its arguments with elements of A.

k-translation provided λ can be realized as the composition of a sequence of k or fewer basic translations.

Translation provided λ is a k-translation for some natural number k.

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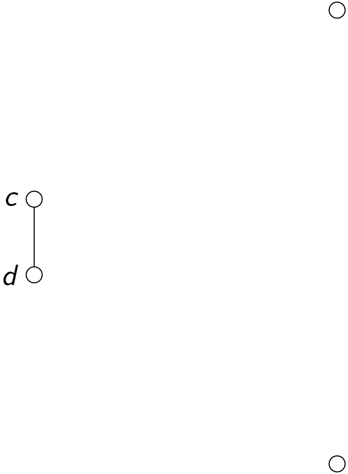
Translation provided λ is a k-translation for some natural number k.

Observe that the identity function is the only 0-translation.

According to Dilworth and Mal'cev

$\langle a, b \rangle \in \mathrm{Cg}^{\mathbf{A}}(c, d)$



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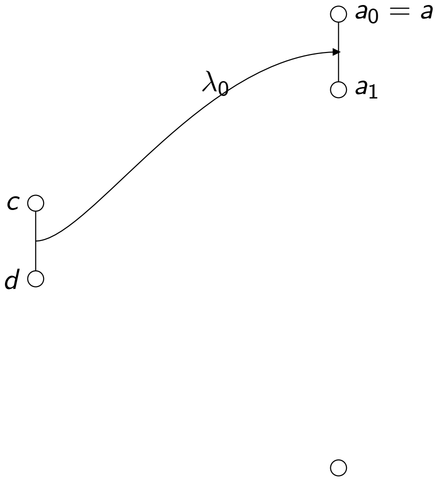

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$\{\lambda_0(c), \lambda_0(d)\} = \{a_0, a_1\}$

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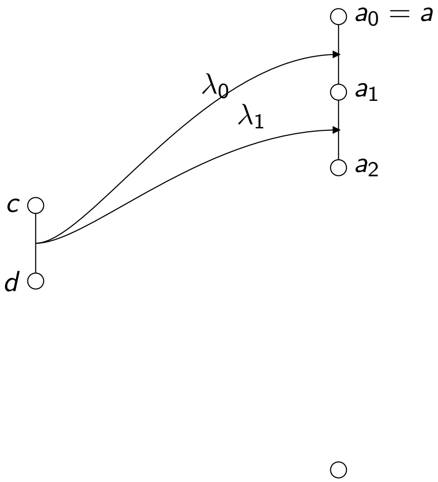

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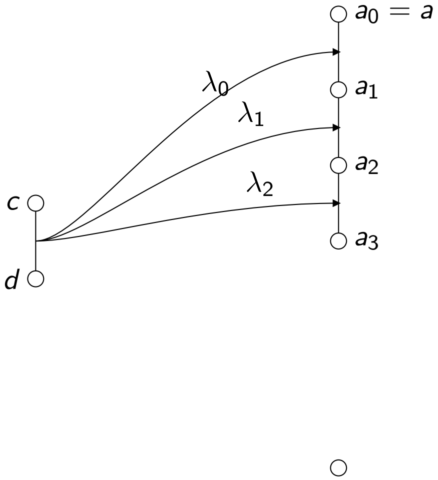

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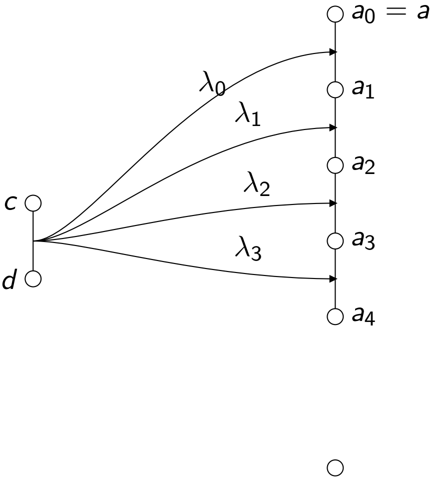

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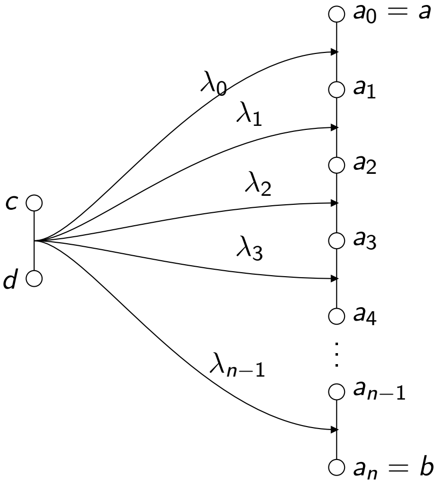
According to Dilworth and Mal'cev



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$$\vdots$$
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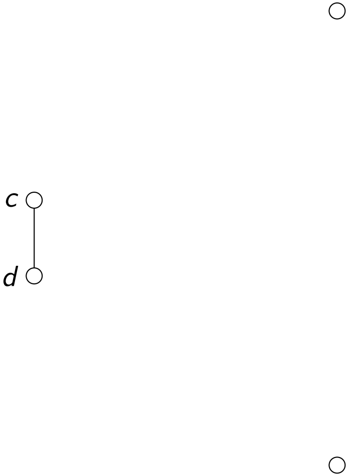
According to Dilworth and Mal'cev



$$\langle a, b\rangle \in \mathrm{Cg}^{\mathbf{A}}(c, d)$$

We write $\{c, d\} \looparrowright^n_k \{a, b\}$
provided
λ_i is a k-translation for each
$i < n$

$\langle a, b\rangle \in \mathrm{Cg}^{\mathbf{A}}(c, d)$
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$\{c, d\} \looparrowright^n_k \{a, b\}$
for some natural numbers k and
n

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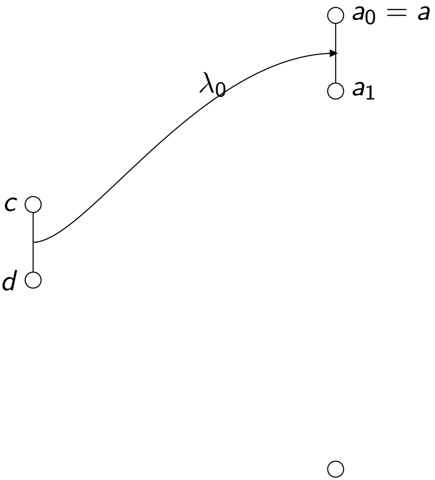

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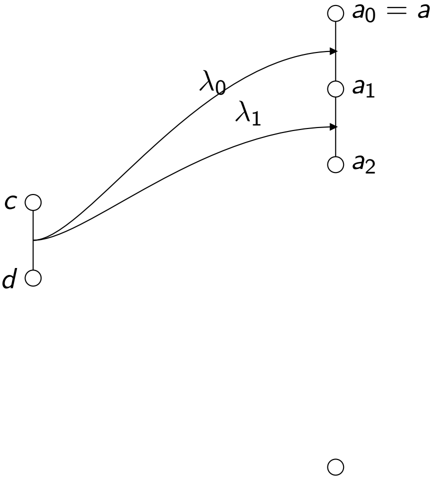

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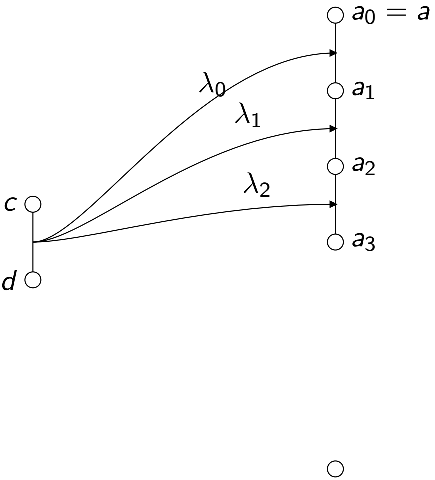

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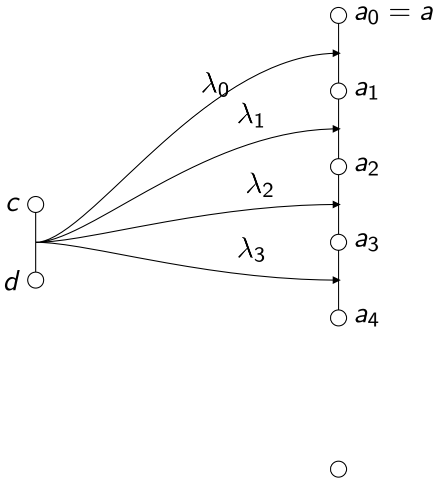

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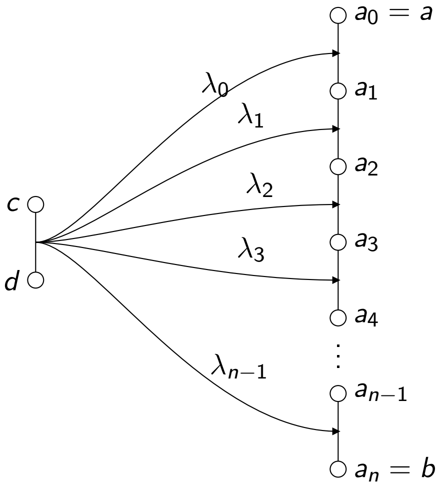

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Outline

Definition

An algebra **A** is **subdirectly irreducible** provided there is distinct elements $p, q \in A$ such that for all $a, b \in A$ with $a \neq b$ $\{a, b\} \looparrowright_k^n \{p, q\}$ for some n and k. In this case $\langle p, q\rangle$ is called a **critical pair**. $\mathcal{K}_{si}$ denotes the class of all subdirectly irreducible algebras belonging to the class $\mathcal{K}$ of algebras.

Definition

An algebra $\mathbf{A}$ is **finitely subdirectly irreducible** if it has at least two elements and for all $a, b, c, d \in A$ with $a \neq b$ and $c \neq d$ there is are two distinct elements p and q of A so that $\{a, b\} \looparrowright_k^n \{p, q\}$ and $\{c, d\} \looparrowright_\ell^m \{p, q\}$ for some natural numbers k, ℓ, m, and n.
$\mathcal{K}_{\mathrm{fsi}}$ denotes the class of all finitely subdirectly irreducible algebras

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Outline

When a Variety $\mathcal{V}$ Has a Finite Residual Bound We Know

- $\mathcal{V}$ is locally finite.

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Definition of Bounded Critical Depth

Definition

A class $\mathcal{K}$ of algebras of the same finite signature is said to have **bounded critical depth** provided there is a natural number ℓ so that for every $\mathbf{A} \in \mathcal{K}_{\mathrm{si}}$ and all $a, b, c, d \in A$ such that $c \neq d$ and $\langle a, b \rangle$ is a critical pair of $\mathbf{A}$ we have $\{c, d\} \looparrowright_{\ell}^{n} \{a, b\}$ for some natural number n.

When a Variety $\mathcal{V}$ Has a Finite Residual Bound We Know

- $\mathcal{V}$ is locally finite.
- $\mathcal{V}_{\text{si}}$ is a finitely axiomatizable elementary class.
- $\mathcal{V}_{\text{si}} = \mathcal{V}_{\text{fsi}}$.
- $\mathcal{V}_{\text{fsi}}$ is a finitely axiomatizable elementary class.
- $\mathcal{V}$ **has bounded critical depth**.

Outline

Theorem (Willard, 2000)

Let $\mathcal{V}$ be a variety of finite signature.
If

- *$\mathcal{V}$ is congruence meet semidistributive.*
- *$\mathcal{V}$ has a finite residual bound.*

then $\mathcal{V}$ is finitely based.

Theorem (Jónsson 1979)

Let $\mathcal{V}$ be a variety of finite signature.
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A New Theorem

Theorem (With Kirby Baker and Ju Wang)

Let $\mathcal{V}$ be a variety of finite signature.
If

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- *$\mathcal{V}_{\mathsf{fsi}}$ is finitely axiomatizable.*
- *$\mathcal{V}$ is locally finite.*
- *$\mathcal{V}$ has bounded critical depth.*

then $\mathcal{V}$ is finitely based.

Outline

Definition

A class $\mathcal{K}$ of algebras of some finite signature has **term-finite principal congruences** provided there is a natural number ℓ so that for every $\mathbf{A} \in \mathcal{K}$ and all $a, b, c, d \in A$ we have

$$\langle a, b\rangle \in \mathrm{Cg}^{\mathbf{A}}(c, d) \text{ if and only if } \{c, d\} \looparrowright^{n}_{\ell} \{a, b\} \text{ for some } n.$$

Corollary

Let $\mathcal{V}$ be a variety of finite signature.
If

- *$\mathcal{V}$ is congruence meet semidistributive.*
- *$\mathcal{V}_{\mathrm{fsi}}$ is finitely axiomatizable.*
- *$\mathcal{V}$ is locally finite.*
- *$\mathcal{V}_{\mathrm{si}}$ has term-finite principal congruences.*

then $\mathcal{V}$ is finitely based.

Definition

A class $\mathcal{K}$ of algebras of some finite signature has **bounded critical diameter** provided there is a natural number ℓ so that for every $\mathbf{A} \in \mathcal{K}_{\mathrm{si}}$ and all $a, b, c, d \in A$ such that $\langle a, b\rangle$ and $\langle c, d\rangle$ are critical pairs we have $\{c, d\} \looparrowright_{\ell}^{n} \{a, b\}$ for some n.

Corollary

Let $\mathcal{V}$ be a variety of finite signature.
If

- *$\mathcal{V}$ is congruence meet semidistributive.*
- *$\mathcal{V}_{fsi}$ is finitely axiomatizable.*
- *$\mathcal{V}$ is locally finite.*
- *$\mathcal{V}_{si}$ is an elementary class.*
- *$\mathcal{V}$ has bounded critical diameter.*

then $\mathcal{V}$ is finitely based.

Example

Let $\mathcal{D}$ be the variety generated by the flat graph algebra **D**. Then

- $\mathcal{D}$ is congruence meet semidistributive.
- $\mathcal{D}$ is locally finite.
- $\mathcal{D}_{si}$ is finitely axiomatizable.
- $\mathcal{D}_{fsi}$ is finitely axiomatizable.
- $\mathcal{D}$ has bounded critical diameter.
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- $\mathcal{D}$ is residually large.
- $\mathcal{D}$ is finitely based.

Most of these items were pointed out by Zoltán Székely (1998) and Dejan Delić (2001). In particular, Delić showed that $\mathcal{D}$ is finitely based by a different method.

Outline

Definition

An elementary formula $\Phi(x, y, z, w)$ is said to **define nontrivial principal meets** in an algebra **A** provided for all $a, b, c, d \in A$ with $a \neq b$ and $c \neq d$

$$\mathrm{Cg}^{\mathbf{A}}(a, b) \cap \mathrm{Cg}^{\mathbf{A}}(c, d) \text{ is nontrivial}$$
$$\text{if and only if}$$
$$\mathbf{A} \models \Phi(a, b, c, d)$$

Definition

An elementary formula $\Phi(x, y, z, w)$ is said to **define nontrivial principal meets** in a class $\mathcal{K}$ of algebras if and only if $\Phi(x, y, z, w)$ defines nontrivial principal meets is every algebra belonging to $\mathcal{K}$.

We regard $\{x, y\} \looparrowright^n_k \{u, v\}$ as an elementary formula with free variables x, y, u and v.

A more elaborate elementary formula is $\{x, y\} \looparrowright^m_\ell \circ \looparrowright^n_k \{u, v\}$.

The elementary formula

$$\exists u, v\,[u \neq v \;\curlywedge\; \{x, y\} \looparrowright^m_\ell \circ \looparrowright^n_k \{u, v\} \;\curlywedge\; \{z, w\} \looparrowright^m_\ell \circ \looparrowright^n_k \{u, v\}]$$

is built from $\{x, y\} \looparrowright^m_\ell \circ \looparrowright^n_k \{u, v\}$.

Definition

We say $\{x, y\} \looparrowright^m_\ell \circ \looparrowright^n_k \{u, v\}$ **supports nontrivial principal meets** provided

$$\exists u, v\, [u \neq v \curlywedge \{x, y\} \looparrowright^m_\ell \circ \looparrowright^n_k \{u, v\} \curlywedge \{z, w\} \looparrowright^m_\ell \circ \looparrowright^n_k \{u, v\}]$$

defines nontrivial principal meets.

Defining Nontrivial Principal Meets

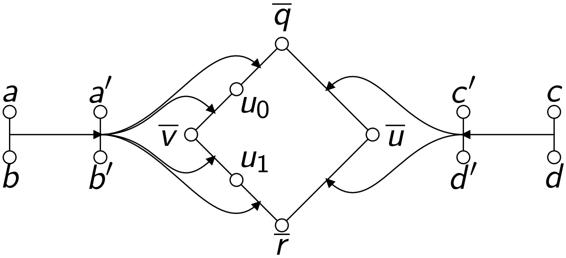
Theorem (With Kirby Baker and Ju Wang)

Let $\mathcal{V}$ be a variety of finite signature.
If

- *$\mathcal{V}$ is congruence meet semidistributive.*
- *$\mathcal{V}$ is locally finite.*
- *$\mathcal{V}$ has critical depth bounded by ℓ.*

then there are natural numbers k and n such that
$\{x, y\} \looparrowright_{\ell}^{1} \circ \looparrowright_{k}^{n} \{u, v\}$ supports nontrivial principal meets in $\mathcal{V}$.

Supporting Nontrivial Principal Meets



Theorem (with Kirby Baker and Ju Wang)

Let $\mathcal{V}$ be a variety with a finite signature. If there are natural numbers $\ell, n,$ and k and an elementary sentence Ψ such that

- *$\mathcal{V}_{\mathrm{fsi}}$ is finitely axiomatizable;*
- *Ψ is true in $\mathcal{V}$;*
- *$\{x, y\} \looparrowright^{1}_{\ell} \circ \looparrowright^{n}_{k} \{u, v\}$ supports nontrivial principal meets in $\mathcal{V}$.*
- *For all algebras $\mathbf{B} \models \Psi$ and all $a, b, c, d \in B$ if $\mathrm{Cg}^{\mathbf{B}}(a, b) \cap \mathrm{Cg}^{\mathbf{B}}(c, d)$ is nontrivial, then there are $q, r \in B$ with $q \neq r$ and there is a natural number m so that $\{a, b\} \looparrowright^{1}_{m} \circ \looparrowright^{n}_{k} \{q, r\}$ and $\{c, d\} \looparrowright^{1}_{m} \circ \looparrowright^{n}_{k} \{q, r\}$;*

then $\mathcal{V}$ is finitely based.

Outline

The Congruence Modular Temptation

Let $\mathcal{V}$ be a variety of finite signature.
If

- $\mathcal{V}$ is congruence modular.
- $\mathcal{V}_{\mathsf{fsi}}$ is finitely axiomatizable.
- $\mathcal{V}$ is locally finite.
- $\mathcal{V}$ has bounded critical depth.
- $\ldots$.

then is $\mathcal{V}$ is finitely based?

The Blatant Temptation

Let $\mathcal{V}$ be a variety of finite signature.
If

-
- $\mathcal{V}_{\mathsf{fsi}}$ is finitely axiomatizable.
- $\mathcal{V}$ is locally finite.
- $\mathcal{V}$ has bounded critical depth.
- ....

then is $\mathcal{V}$ is finitely based?


W Taylor