Sample Examination Problems
Mathematics 141
Professor George McNulty

Problem 0.

a. Let $F(x) = \sqrt{(x-1)^{-1}}$. Describe the natural domain of $F(x)$.

b. Let $G(t) = \dfrac{t}{t-5}$. Simplify $\frac{G(a+h)-G(a)}{h}$

c. Let $H(x) = \ln\sqrt{x^2+2}$. Find four functions $f(x), g(x), h(x),$ and $k(x)$ so that

$$H(x) = (f \circ g \circ h \circ k)(x)$$

Problem 1.

a. Demonstrate that $\sin^2 x + \tan^2 x = \sec^2 x - \cos^2 x$

b. Demonstrate that $\log_a b \log_b c = log_a c$ for positive numbers $a, b,$ and c such that $a \neq 1 \neq b$.

Problem 2 (Core).

In each part below, use the definition of limit to verify the statement.

a. $\lim\limits_{x\to 2}(2x-1) = 3$

b. $\lim\limits_{x\to 1}(x^2-1) = 0$

Problem 3.

In each part below, use our theorems to discover the limit. Be sure to show all your work.

a. $\lim\limits_{x\to 1}(x^2+3x-2)$

b. $\lim\limits_{x\to 2}\dfrac{1}{x^3-1}$

c. $\lim\limits_{x\to 3}\dfrac{x^2-x-6}{x-3}$

Problem 4.

In each part below, use our theorems about derivatives to discover the derivative of the function given. Be sure to show all your work.

a. Let $F(x) = (2x+1)(3x+2)$.

b. Let $G(x) = \dfrac{2x+1}{3x+2}$.

c. Let $H(x) = (2x+1)^{100}$.

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Problem 5 (Core).

In each part below, use our theorems about derivatives to discover the derivative of the function given. Be sure to show all your work.

a. Let $F(x) = \left(\dfrac{x-1}{x+1}\right)^5$.

b. Let $G(x) = \sqrt{\dfrac{x^2-1}{x^2+1}}$.

c. Let $H(x) = x(2x+1)^{100}$.

Problem 6 (Core).

In each part below, use the definition of the derivative to discover the derivative of the function given. Be sure to show all your work.

a. Let $F(x) = 3x + 1$.

b. Let $G(x) = \dfrac{1}{2x+1}$.

Problem 7.

Explain why every positive real number has a positive square root.

[Hint: Consider the squaring function $f(x) = x^2$.]

Here is another sample of the same sort of problem:

Explain why any line segment with one endpoint inside the unit circle about the origin and the other outside the same unit circle must have a point which lies on the unit circle.

Problem 8 (Core).

a. Calculate the derivative of $\cos^2(2x+1)$.

b. Calculate the derivative of $\sec x + \tan x$.

c. Calculate the derivative of $e^{2x} + \ln 3x$.

d. The equation $xy^2 = 1$ describes a curve in the plane. Find an equation which describes the line tangent to this curve at the point $(1, -1)$.

Problem 9 (Core).

a. Consider the curve described by the equation $y = x^3 - 3x^2 - 9x + 1$. Find the intervals on which this curve is increasing, on which it is decreasing, on which it is concave upward, and only which it is concave downward. Locate all the local minima and local maxima, and all the points of inflection. Finally, sketch the curve.

b. Consider the curve described by $y = \dfrac{x^2+3}{x-1}$. Find equations describing all the assymptotes of this curve. Find all the local minima and local maxima. Finally, sketch this curve.

Problem 10.

A spherical ball of ice is melting at the rate of 3 cubic feet per minute. Determine how fast the surface area of the ball is decreasing when the radius is 5 feet.

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PROBLEM 11.

What are the dimensions of the cylindrical can of volume 16π cubic centimeters which has the smallest surface area? Be sure to justify your answer fully.

PROBLEM 12 (CORE).

In each part below, find the area of the region described.

a. The region between the graph described by $y = 4 - x^2$ and the X-axis between the lines described by $x = -2$ and $x = 2$.

b. The region bounded between the curves described by $y = x^4 - 1$ and $y = 1 - x^2$.

PROBLEM 13.

In each part below, find the length of the curve described.

a. The curve described by $y = 4x^{3/2} + 1$ from $x = 0$ to $x = 9$.

b. The curve described by:

$$\begin{aligned} x &= \sin t - \cos t \\ y &= \sin t + \cos t \\ \pi/4 \leq t &\leq \pi/2 \end{aligned}$$

PROBLEM 14.

In each part below, find the volume of the solid described. Use calculus to justify your answers.

a. The solid obtained by revolving an equilateral triangle of side s about one of its sides.

b. A straight hole is bored through a very large wooden sphere. The drilling starts at the "north pole" of the sphere, proceeds through the center, and emerges at the "south pole" of the sphere. This hole is so wide that when its length is measured (from the rim at the north directly to the rim at the south) it turns out to be 6 inches. What remains, apart from the sawdust, is a ring-shaped solid. What is the volume of this solid?

PROBLEM 15.

Gasoline at a service station is stored in a cylindrical tank buried on its side, with the highest part of the tank 5 feet below the surface. The tank is 10 feet long and 6 feet in diameter. A cubic foot of gasoline weighs 45 pounds. Assume that the filler cap of each automobile gastank is 2 feet above the ground. How much work is done in emptying all the gasoline from the buried tank (which is initially full) into automobiles? [Hint: $\int_{-3}^{3} 2\sqrt{9 - y^2}dy$ gives the area of a circle of radius 3. This fact may prove useful at some point in your solution.]