Instructions: This quiz is an individual effort. Answer each question to the best of your ability. You are not permitted to use any resource besides your own knowledge to complete this quiz. The total points is 20. You may do work on a seperate piece of paper, but all answers must be written on the quiz and you must turn in your work. This means your work must be organized for the instructor to follow your thought process.
If you are unable to complete a question, whether it be due to time or not, write out what you would do or what you know relates to the question. This shows me you at least have knowledge on the subject matter and some knowledge on how to approach the problem, but are stuck on the execution.

1. a. Prove that $\mathbb{Z}_6$ is an abelian group under addition.

   b. (Challenge problem) Explain or show why $\mathbb{Z}_6$ is not a group under multiplication but $\mathbb{Z}_{11}$ is.

2. Let $A = \{x \in \mathbb{Z} \mid 0 \leq x \leq 5\}$ and define the relation $a \sim b$ as the following:

$$a \sim b \Leftrightarrow a \equiv b \pmod 4 \text{ for } A \text{ to } A, \text{ or}$$

a and b have the same remainder when divided by 4.

a. Find the matrix of the relation.

b. Find the digraph of the relation.

c. Determine if this is an equivalence relation through proof or contradiction. (The above may even be enough to say this or not say this, but you must explain that.)

3. Let f, g and h be the following possible functions:

$$f : \mathbb{R} \to \mathbb{R} \text{ such that } f(x) = \frac{3x+1}{1-2x}$$
$$g : \mathbb{R} \to \mathbb{R}^+ \cup \{0\} \text{ such that } g(x) = x^4$$
$$h : \mathbb{R} \to \{0,1\} \text{ such that } h(x) = \chi_{\mathbb{Z}}(x) = \left\{ \begin{array}{ll} 0 & x \in \mathbb{Z} \\ 1 & x \notin \mathbb{Z} \end{array} \right\}$$

a. Are these all functions? Explain.

b. Determine which functions are onto, one-to-one, both or neither. Some proof or explanation is required.

c. Determine which functions have an inverse that is a function. If they have an inverse, find it. If they do not, explain why.