Name: Quiz 4 Solutions **Quiz 4**

Instructions: This quiz is an individual effort. Answer each question to the best of your ability. You are not permitted to use any resource besides your own knowledge to complete this quiz. The total points is 20. You may do work on a seperate piece of paper, but all answers must be written on the quiz and you must turn in your work. This means your work must be organized for the instructor to follow your thought process.

If you are unable to complete a question, whether it be due to time or not, write out what you would do or what you know relates to the question. This shows me you at least have knowledge on the subject matter and some knowledge on how to approach the problem, but are stuck on the execution.

1. For the below recursive sequences, find the explicit formula starting at $n = 1$.

   a. $a_n = (n+1)a_{n-1}$, $a_1 = 4$.

   $a_n = (n+1)n\,a_{n-2} = (n+1)n(n-1)\,a_{n-3} = \dots = (n+1)(n)\cdots(n-(n-3))\,a_{n-(n-1)}$

   $= (n+1)\cdots 3\,a_1$

   $= \frac{(n+1)!}{2}(4) = 2(n+1)! \implies a_n = 2(n+1)!$

   b. $a_n = 5a_{n-1} - 6a_{n-2}$, $a_1 = 6$, $a_2 = 6$.

   $x^2 - 5x + 6 = 0$

   $(x-3)(x-2) = 0$

   $x = 3, 2$

   $a_n = (3)^n u + 2^n v$

   $6 = a_1 = 3u + 2v \implies v = 3 - \frac{3}{2}u$

   $6 = a_2 = 9u + 4v \implies 6 = 9u + 4(3 - \frac{3}{2}u)$

   $= 9u + 12 - 6u$

   $-6 = 3u$ $\quad u = -2 \implies v = 3 - \frac{3}{2}(-2) = 6$

   $a_n = 3^n(-2) + 6 \cdot 2^n$

2. A multiple choice test has 10 questions, each with 4 possible answers. Unlike all of you in this class, a student guesses on all 10 questions.

   a. What is the probability the student gets all 10 right?

   Total options: 4^{10}

   $E = \{\text{all right}\}$

   $|E| = 1$

   $p(E) = 1/4^{10}$

   b. Find the probability they get at least 1 question right.

   $p(\text{all wrong}) = 3^{10}/4^{10}$

   $p(\text{at least 1 right}) = 1 - 3^{10}/4^{10}$

3. Let A be a set of 20 distinct integers chosen from the set $\{1, 4, 7, \ldots, 100\}$.

a. Prove there must be 2 distinct integers in A whose sum is 104.

Pf: Lets look at all possible pairs who add to 104. (4,100), (7,97), (10,94), (13,91), (16,88), (19,85), (22,82), (25,79), (28,76), (31,73), (34,70), (37,67), (40,64), (43,61), (46,58), (49,55), (52,52). This is 17 pairs. 1 is the only unused number. Thus, 18 choices can be made before having to choose a pair. Since we are choosing 20, we will have a pair.

b. Above you've shown that there is a pair of integers in A whose sum is 104. Now determine the minimum number of pairs that can be in A that add up to 104.

2! From above, 18 is the max for non-pairs. Thus, 2 pairs will form given 20 choices.

4. John is a gambling computer science major, but he also knows that math can help him gamble smarter. Thus, John is learning probability for a new game he found online where he pays \$5 to play. The game goes as follows: The computer randomly generates a number 1 through 7. If you get a 6 or higher you win \$10. If you get a 4 or higher you at least get your money back. If you get below a 4 you lose. Being the computer science major he is, John made his own program based off some information he saw from the computer online and was able to determine the following:

(a) $P(7) = P(1)$

(b) $P(6) = P(4) = P(2)$

(c) $P(5) = P(3)$ (3) $\Rightarrow 2p(5) = 3p(7)$

(d) $2P(3) = 3P(1)$

(e) $P(\text{more than } 4) = \frac{3}{8}$ (1) $\Rightarrow p(5) + p(6) + p(7) = 3/8 \Rightarrow p(1) + p(2) + p(3) + p(4) = 5/8$

(2) $p(7) + 2p(6) + p(5) = 5/8$

John knows you are an expert with probability and asks you to tell him the following:

a. The probability for each outcome. (Feel free to use another piece of paper and turn that in)

b. Let expected payout be the following: $E = \sum_{n=1}^{y} P(n) \cdot (\text{earnings})$ where earnings is the earned amount minus the cost to play. Determine E for John and tell him whether he should play or not.

(a) (2) − (1) $\Rightarrow p(6) = 1/4 = p(2) = p(4)$.

(3) ⇒ (1) $\Rightarrow p(5) + 1/4 + \frac{2}{3}p(5) = 3/8 \Rightarrow \frac{5}{3}p(5) = 1/8 \Rightarrow p(5) = 3/40 = p(3)$

Thus $2(3/40) = 3p(7) \Rightarrow \frac{2}{20} = 3p(7) \Rightarrow p(7) = \frac{1}{20} = p(1)$

(b) $E = (-5)(p(1) + p(2) + p(3)) + 0(p(4) + p(5)) + 5(p(6) + p(7))$

$= (-5)\left(\frac{1}{20} + \frac{1}{4} + \frac{3}{40}\right) + 5\left(1/4 + \frac{1}{20}\right)$

$= (-5)(15/40) + 5(6/20) = -\frac{5 \cdot 15}{40} + \frac{5 \cdot 12}{40} = 5 \cdot \left(\frac{-3}{40}\right) = \$ -\frac{3}{8} < 0$ no play!