# Quiz 3 Better Solutions

(1) If we assume these 4 rows to be the only cases, we notice the only difference is the last column is different. Why? Patrick says $F \Rightarrow T$ is F and spongebob says $F \Rightarrow T$ is T. Thus, Spongebob is right since once the 1st part of the implication is False, it does not matter what the 2nd part is; it will always be possible. Implication requires the 1st part to be T to say anything about the 2nd.

(2)

| $p$ | $q$ | $p \Rightarrow q$ | $\sim(p \Rightarrow q)$ | $\sim p$ | $\sim q$ | $p \wedge \sim q$ | $q \wedge \sim p$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F | F |
| T | F | F | T | F | T | T | F |
| F | T | T | F | T | F | F | T |
| F | F | T | F | T | T | F | F |

X

(3) (a) Follows exactly from $\sqrt{2}$ irrational!

Pf: Suppose $\sqrt{3}$ is rational for a contradiction. Then $\sqrt{3} = \frac{a}{b}$ $a, b \in \mathbb{Z}, b \neq 0, \gcd(a,b)=1$.

Then $3 = \frac{a^2}{b^2} \Rightarrow 3b^2 = a^2$. Thus $3 \mid a^2$ and $3 \mid a$. Therefore $a = 3k, k \in \mathbb{Z}$. Thus,

$\sqrt{3} = \frac{a}{b} = \frac{3k}{b} \Rightarrow 3 = \frac{(3k)^2}{b^2} \Rightarrow 3b^2 = 9k^2 \Rightarrow b^2 = 3k^2$. Thus, $3 \mid b^2$ and $3 \mid b$.

Therefore $\gcd(a,b) \geq 3$. ($\Rightarrow\Leftarrow$ of $\gcd(a,b)=1$). Therefore, $\sqrt{3}$ must be irrational $\square$

(3) (b) $n$ is not divisible by 3 means it has remainder 1 or 2 when divided by 3. Thus, $n = 3k+1$ or $3k+2$ $k \in \mathbb{Z}$.

Note: $n^2 \equiv 1 (3)$ means $n^2 = 3\ell + 1$ $\ell \in \mathbb{Z}$. or remainder 1.

Pf: case 1: Suppose $n = 3k+1$ $k \in \mathbb{Z}$. Then $n^2 = (3k+1)^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1$

Thus, $n^2 \equiv 1 \pmod 3$.

case 2: Suppose $n = 3k+2$, $k \in \mathbb{Z}$. Then $n^2 = (3k+2)^2 = 9k^2 + 12k + 4 = 9k^2 + 12k + 3 + 1$
$= 3(3k^3 + 4k + 1) + 1$

Thus, $n^2 \equiv 1 (3)$. Therefore we are done $\square$

(c) The contrapositive of this statement is if $n$ is not divisible by 3 then $n^2$ is not divisible by 3. (or in other words, not remainder 0 but remainder 1 or 2 when divided by 3)

Pf: This is what part b is saying. Thus, by contrapositive we are done $\square$

(4)

Pf: By induction.

nothing crazy here!

Base case: $n=1$. Then $a_1 = 0$ ✓.

~~Ind. Step~~: Suppose true for $n \geq k$. That is $a_1 \cdots a_k = 0$ then for some $1 \leq i \leq k$, $a_i = 0$

we show true for ~~[illegible]~~ $n = k+1$. That is $a_1 \cdots a_k \cdot a_{k+1} = 0$ then for some $1 \leq i \leq k+1$ $a_i = 0$.

There are 2 cases to consider, either ~~[illegible]~~ $1 \leq i \leq k$ or $i = k+1$. If $1 \leq i \leq k$, Then we know true from inductive hypothesis! If not, then it must be the case that $a_{k+1} = 0$. Therefore, we are done $\square$