

Instructions: This quiz is an individual effort. Answer each question to the best of your ability. You are not permitted to use any resource besides your own knowledge to complete this quiz. The total points is 20. You may do work on a separate piece of paper, but all answers must be written on the quiz and you must turn in your work. This means your work must be organized for the instructor to follow your thought process.

If you are unable to complete a question, whether it be due to time or not, write out what you would do or what you know relates to the question. This shows me you at least have knowledge on the subject matter and some knowledge on how to approach the problem, but are stuck on the execution.

1. Spongebob is trying to show Patrick that if he goes jellyfish hunting and Patrick is with him then he will catch a jellyfish or Patrick will catch a jellyfish. To do this Spongebob will show this is a tautology with a truth table labeled S. Thus, Spongebob assumes the p = Spongebob going jellyfishing, q = Patrick going jellyfishing, r = Spongebob catches a jellyfish, and s = Patrick catches a jellyfish are all statements. Patrick disagrees and thinks the table should look more like the below labeled P and, thus, is not a tautology.

S						
p	q	r	s	$p \wedge q$	$r \vee s$	$(p \wedge q) \Rightarrow (r \vee s)$
T	T	T	T	T	T	T
T	F	T	F	F	T	T
F	T	F	T	F	T	T
F	F	F	F	F	F	T

P						
p	q	r	s	$p \wedge q$	$r \vee s$	$(p \wedge q) \Rightarrow (r \vee s)$
T	T	T	T	T	T	T
T	F	T	F	F	T	F
F	T	F	T	F	T	F
F	F	F	F	F	F	T

To settle the argument, they run to Squidward (You) to ask who is right. Write down who Squidward would agree with and why. Then write exactly how you think Squidward would say it to Spongebob and Patrick. (Please have fun with this!)

2. Determine which is/are true via truth table(s):

(a)

$$\sim (p \Rightarrow q) \equiv p \wedge \sim q$$

(b)

$$\sim (p \Rightarrow q) \equiv q \wedge \sim p.$$

3. Prove the following:

a. $\sqrt{3}$ is irrational.

b. If n is not divisible by 3 then $n^2 \equiv 1 \pmod{3}$ (in other words has remainder 1 when divided by 3). Hint: If n is not divisible by 3, what possible remainders can it have?

c. Use the above to prove: If n^2 is divisible by 3 then n is divisible by 3.

4. Use induction to prove the following: If a_1, a_2, \dots, a_n are real numbers and $a_1 \cdot a_2 \cdots a_n = 0$ then $a_i = 0$ for some $1 \leq i \leq n$. Hint: Your inductive step should have 2 cases.