Solutions

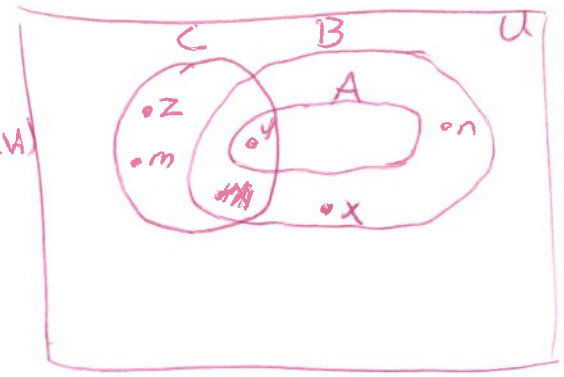
Name: **Quiz 2**

Instructions: This quiz is an individual effort. Answer each question to the best of your ability. You are not permitted to use any resource besides your own knowledge to complete this quiz. The total points is 20. You may do work on a seperate piece of paper, but all answers must be written on the quiz and you must turn in your work. This means your work must be organized for the instructor to follow your thought process.
If you are unable to complete a question, whether it be due to time or not, write out what you would do or what you know relates to the question. This shows me you at least have knowledge on the subject matter and some knowledge on how to approach the problem, but are stuck on the execution.

1. Draw a Venn Diagram for the sets A, B, and C such that the below information is true. Use U to denote the universal set and a black dot labeled accordingly for each element.

multiple answers!

   (a) $A \subseteq B$
   (b) $x \in (B \cup C)$
   (c) $y \in (A \cap B \cap C)$
   (d) $z \in (A \Delta C)$ = $z \in (A \cup C) \cap (C \cup A)$
   (e) $m \in (\overline{B} \cap C)$
   (f) $n \in (B - A)$



2. Let A be the following set:

$$A = \{x | x^2 - x < 20, x \in \mathbb{Z}^+\}$$

$= \{1, 2, 3, 4\}$ $5^2 - 5 = 20 \not< 20$

Determine the following:

   a. $|A| = 4$

   b. Number of subsets of A. $2^4 = 16$

   c. Number of proper subsets of A. $16 - 1 = 15$

   d. List all proper subsets of A.
   $\{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}$
   $\{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}$

   e. Any additional condition on A so that $A = \emptyset$.
   $x^3 \geq 125$ many answers

3. Let $A = \{2, 5, 10, 17, 26, \ldots\}$ be an ordered set. Write a formula for the nth term of a sequence that is represented by the ordering in A. (Please start with $n = 1$)
Hint: Think powers.

$n \geq 1$, $a_n = n^2 + 1$

4. Let $m = 97$ and $n = 20$.

a. Find $\gcd(m, n)$ using the Euclidean Algorithm.

$97 = 4 \cdot 20 + 17$ $\quad 3 = 1 \cdot 2 + 1$
$20 = 1 \cdot 17 + 3$ $\quad 2 = 2 \cdot 1 + 0$
$17 = 5 \cdot 3 + 2$

b. Determine $\text{lcm}(m, n)$. (Please express as a product of integers)

$20 = 2^2 \cdot 5$ $\quad \text{lcm}(20, 97) = 20 \cdot 97$
$97 = 97^1$

c. Use the Extended Euclidean Algorithm to find a linear combination of m and n for $\gcd(m, n)$.

$1 = 3 - 1 \cdot 2$
$1 = 3 - 1(17 - 5 \cdot 3) = 3 - 1 \cdot 17 + 5 \cdot 3 = 6 \cdot 3 - 1 \cdot 17$
$1 = 6(20 - 1 \cdot 17) - 1 \cdot 17 = 6 \cdot 20 - 7 \cdot 17$

$1 = 6 \cdot 20 - 7 \cdot 17$
$1 = 6 \cdot 20 - 7(97 - 4 \cdot 20)$
$1 = -7 \cdot 97 + 34 \cdot 20$

d. What does (a) tell you about the prime factorization of m?

It is prime to 20 (in fact it is prime)
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5. Let A and B be 2 x 2 matrices such that the elements of the diagonals are all 1.

Ex: $\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$.

If $AB = BA$, what condition on the elements of A and B must be true?

Hint: Consider the general forms: $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}$.

$AB = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}\begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix} = \begin{pmatrix} 1+ad & c+a \\ b+d & 1+bc \end{pmatrix}$

$BA = \begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} = \begin{pmatrix} 1+bc & a+c \\ d+b & ad+1 \end{pmatrix}$

$\Rightarrow 1 + ad = 1 + bc$

$ad = bc$