Homework 4 Solutions

3.3

(2) (1, 12) (2, 11) (3, 10) (4, 9) (5, 8) (6, 7)

Since 6 pairs are here the 7^{th} number picked must be in one of these pairs. Thus, 2 will have to add to 13.

(4) 0, 1, 2, 3, 4, 5, 6, ~~7~~ are all remainders of 7.

Since there are 7 choices, ~~the~~ ~~first~~ assigning 7 of the integers to ~~each~~ different remainders leaves 1 integer that must be paired up.

(10) $a_1 + a_2 + \cdots + a_6 = b$ is integer between a_i 1–15.

$$_{15}C_6 = \frac{15!}{(15-6)!\,6!} = \frac{15!}{9!\,6!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 5 \cdot 7 \cdot 11 \cdot 13 = 5005$$

possible sums: $\underbrace{75 - 21}_{\text{max} \quad \text{min}} + 1 = 55$ total possible sums

By P.P. $\lceil 5005/55 \rceil = 91$ ways to chose 6 #'s ✓

(14) ~~lowest sum~~ (1, 20) (2, 19) (3, 18) (4, 17) (5, 16) (6, 15)

(7, 14) (8, 13) (9, 12) (10, 11) It is possible!

3.4

(2) $S = \{TTT, TTH, THT, THH, HTT, HHT, HTH, HHH\}$

(4) $S = \{ab, ac, ad, bc, bd, cd\}$

(10) (a) $E = \{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (1,5), (2,5)$
$(3,5), (4,5)_{(5,5)} (6,5)\}$

(b) $E = \emptyset$

(c) $E = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4),$
include (a_1, a_2) flip of same $(3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$

(d) $E = \{(6,6), (6,5), (6,4), (6,3), (5,6), (5,5), (5,4), (4,6), (4,5)\}$

(22) 1, 2, and 3 equally likely   4 and 5 equally likely

Prob outcome is > 2 is $\frac{1}{2}$ $\Leftrightarrow$ ≤ 2 is $\frac{1}{2}$

$P(4) + P(5) = 2P(5)$ we know $P(3) + P(4) + P(5) = P(3) + 2P(5) = \frac{1}{2}$

$P(1) + P(2) = 2P(1) = \frac{1}{2}$
$P(1) = \frac{1}{4} = P(2) = P(3)$ → $\frac{1}{4} + 2P(5) = \frac{1}{2}$
$2P(5) = \frac{1}{4}$
$P(5) = \frac{1}{8} = P(4)$

(2.6) (a) $E = \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4),(4,1)(4,2),(4,3),(4,4)$
$(4,5)(4,6)\}$

$|E| = 11$

$|S| = 36 = 6 \cdot 6$

$P(E) = 11/36$

(b) 2, 3, 5 $|E| = |\{2 \text{ tossed}\}| + |\{3 \text{ tossed}\}| + |\{5 \text{ tossed}\}|$

$- |\{2 \text{ and } 3 \text{ tossed}\}| - |\{2 \text{ and } 5 \text{ tossed}\}| - |\{3 \text{ and } 5 \text{ tossed}\}|$

$|E| = 11 + 11 + 11 - 2 - 2 - 2 = 27$

$P(E) = 27/36 = 9/12 = 3/4$

(c) $E = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1),\}$

$|E| = 6$

$P(E) = 6/36 = 1/6$

(d) $E = \{(6,6),(6,5),(6,4),(6,3),(6,2)(6,1),(5,6),(5,5),(5,4)(5,3),(5,2)$
$(4,6),(4,5),(4,4),(4,3),(3,6),(3,5),(3,4),(2,6),(2,5),(1,6)\}$

$P(E) = 21/36 = 7/12$

(30) (a) $E = \{(1,2,2), (3,2,2)$

$|E| = |\{\text{die } 1 \text{ not } 2 \text{ but all others } 2\}| + |\{\text{die } 2 \text{ is not } 2, \text{ all other } 2\}| + |\{\text{die } 3 \text{ not } 2, \text{ all others } 2\}|$

$= 5 + 5 + 5$

$= 15$

$p(E) = 15/6^3 = 15/216 = 5/(6^2 \cdot 2) = 5/72$

(b) $|E| = |\{\text{die } 1 = 1, \text{ die } 2 = 2 \text{ and die } 3 > 2\}| + |\{\text{die } 1 = 1 \text{ die } 2 = 3 \text{ die } 3 > 3\}|$

$+ |\{\text{die } 1 = 1, \text{ die } 2 = 4 \text{ die } 3 > 4\}| + |\text{" \quad " die } 3 > 5\}|$

$+ |\{\text{die } 1 = 2 \text{ die } 2 = 3 \text{ die } 3 > 3\}| + \cdots$

$+ |\{\text{die } 1 = 4 \text{ die } 2 = 5 \text{ die } 3 = 6\}|$

$= 4 + 3 + 2 + 1 + 3 + 2 + 1 + 2 + 1 + 1$

$= 4 + 2(3) + 3(2) + 4(1) = 8 + 12 = 20$

$p(E) = 20/6^3 = 5/(6 \cdot 3^2) = 5/54$

(c) $|E| = |\{\text{die } 1 = 3\}| + |\{\text{die } 2 = 3\}| + |\{\text{die } 3 = 3\}| - |\{\text{die } 1 \text{ and die } 2 = 3\}|$

$- |\{1 \text{ and } 3\}| - |\{2 \text{ and } 3\}| + |\{\text{all } 3\}|$

$= 36 + 36 + 36 - 6 - 6 - 6 + 1 = 91 \quad p(E) = 91/216$

(d) $|E| = |\{\text{1st roll} = 3 \text{ others not}\}| + |\{\text{2nd} = 3 \text{ others not}\}| + |\{\text{3rd} = 3 \text{ others not}\}|$

$+ |\{\text{none } 3\}|$

$= 25 + 25 + 25 + 125 = 200$

$200/216 = 100/108 = 50/54 = 25/27$ or $1 - (a) - 1/216$

$\frac{216}{216} - \frac{15}{216} - \frac{1}{216} = 200/216$

(e) no 3's?

$P(E) = 1 - P(\text{at least } 1\ 3) = \frac{216 - 91}{216} = \frac{200 - 75}{216} = \frac{125}{216}$

$P(\text{at least } 1\ 3) = 91/216$

3.5

(2) yes deg 2

(6) Not

(18) $b_n = 5(5b_{n-2} + 3) + 3$ $\quad b_1 = 2$

$= 5^2 b_{n-2} + 5 \cdot 3 + 3 = 5^2(5b_{n-3} + 3) + 5 \cdot 3 + 3$

$\vdots$

$b_n = 5^{n-1} b_{n-(n-1)} + \underbrace{5^{n-2} \cdot 3 + 5^{n-3} \cdot 3 + \cdots + 3}_{\text{geo series}}$

$= 5^{n-1} b_1 + 3\left(\frac{5^{n-1} - 1}{5 - 1}\right) = 5^{n-1} \cdot 2 + \frac{3}{4}(5^{n-1} - 1)$

$= \boxed{\frac{11}{4} \cdot 5^{n-1} - \frac{3}{4}}$

(12) $g_n = n g_{n-1}$ $\quad g_1 = 6$

$g_n = n(n-1 g_{n-2}) = n(n-1)(n-2 g_{n-3})$

$\vdots$ $\quad n-(n-2)$

$= n(n-1)(n-2)\cdots (2) g_{n-(n-1)}$

$= n! g_1$

$= 6n!$

(14) $b_n = -3b_{n-1} - 2b_{n-2}$ $\quad b_1 = -2,\ b_2 = 4$

$x^2 + 3x + 2 = (x+2)(x+1)$

roots $x = -2,\ x = -1$

$\Rightarrow$ $b_n = u(-2)^n + v(-1)^n$

$b_1 = -2 = u(-2)^n + v(-1)^n$ $\qquad b_2 = 4 = u(-2)^2 + v(-1)^2$

$-2 = -2u - v$ $\qquad 4 = 4u + v$

$v = 4 - 4u$

$-2 = -2u - (4 - 4u)$

$-2 = 2u - 4$

$\Rightarrow u = 1 \Rightarrow v = 0 \Rightarrow b_n = (-2)^n$

(18) $g_n = 2g_{n-1} - 2g_{n-2}$ $\quad g_1 = 1$ $\quad g_2 = 4$

$x^2 - 2x + 2$ Quadratic formula

$\frac{2 \pm \sqrt{4 - 4(2)}}{2} = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$

$g_n = u(1+i)^n + v(1-i)^n$

$g_1 = 1 = u(1+i) + v(1-i)$

$g_2 = 4 = u(1+i)^2 + v(1-i)^2$

$4 = 2iu - 2iv$

$2 = iu - iv$

$u = \frac{2 + iv}{i} = (2 + iv)(-i)$

$g_1 = 1 = (2+iv)(1+i) + v(1-i)$

$1 = (-i)(2 + 2i + iv - v) + v - iv$

$1 = -2i + 2 + v + \cancel{iv} + v - \cancel{iv}$

$2v \Rightarrow -1 + 2i = 2v$ $\quad \frac{-1+2i}{2} = v$ $\quad \Rightarrow u = \frac{-1-2i}{2}$