Instructions: This quiz is closed book, closed note, and an individual effort. **Show all work to receive full credit.** Unless the question specifies, you should provide an exact answer. If you get stuck, please attempt to explain what you want to do. This may give more partial credit.

WRITE THIS PARAGRAPH ON WHAT YOU SUBMIT ALONG WITH A SIGNATURE AND DATE.
I, ____________________, will not under any circumstance use an online source, my peers, my notes, or any other resource besides my own knowledge to complete this quiz. I will show all my work to demonstrate my knowledge on the topic.

1. Determine if the following transformations are linear transformations from $\mathbf{R}^2$ to $\mathbf{R}^2$. Justify your answer.

   a. $T\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) = \begin{bmatrix} v_1 + v_2 \\ v_2 \end{bmatrix}$

   Solution 1. We check that $T(k\mathbf{v} + \ell\mathbf{u}) = kT(\mathbf{v}) + \ell T(\mathbf{u})$.
   $T(kv_1 + \ell u_1, kv_2 + \ell u_2) = (kv_1 + \ell u_1 + kv_2 + \ell u_2, kv_2 + \ell u_2) = k(v_1 + v_2, v_2) + \ell(u_1 + u_2, u_2) = kT(\mathbf{v}) + \ell T(\mathbf{u})$

   b. $T\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) = \begin{bmatrix} v_1 + 1 \\ 2v_1 \end{bmatrix}$

   Solution 2. We check that $T(k\mathbf{v} + \ell\mathbf{u}) = kT(\mathbf{v}) + \ell T(\mathbf{u})$.
   $T(kv_1 + \ell u_1, kv_2 + \ell u_2) = (kv_1 + \ell u_1 + 1, 2kv_2 + 2\ell u_2) \neq kT(\mathbf{v}) + \ell T(\mathbf{u})$ since the $+1$ cannot be distributed amongst both vectors!

2. Let T be linear transformation from $\mathbf{R}^3$ to $\mathbf{R}^2$. Take the standard basis for $\mathbf{R}^3$ and let

   $\left\{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}\right\}$ be the basis for $\mathbf{R}^2$. Suppose $T(1,0,0) = (2,0)$, $T(0,1,0) = (2,1)$,

   and $T(0,0,1) = (-1,1)$.

   a. Find the matrix representation A under these bases for the linear transformation T.

   Solution 3. $A = \begin{bmatrix} 0 & 1 & 1 \\ -2 & 0 & 3 \end{bmatrix}$.

   b. Calculate $T(2,1,0)$ in two ways, first using the linearity of T and second using the matrix A.

   Solution 4. Way 1: $T(2,1,0) = T(2(1,0,0) + (0,1,0) + 0(0,0,1)) = 2T(1,0,0) + T(0,1,0) + T(0,0,1) = (4,0) + (2,1) = (6,1)$

Way 2: $A\begin{bmatrix}2\\1\\0\end{bmatrix} = \begin{bmatrix}1\\-4\end{bmatrix}$. Thus, $T(2,1,0) = (2,1) - 4(-1,0) = (6,1)$.

3. Suppose $T(M) = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} M \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$. (Hint for this entire problem: what dimension does M have to have? This should be a good start!)

   a. Find a matrix with $T(M) \neq 0$.

   Solution 5. $\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$.

   b. Describe all matrices with $T(M) = 0$. What do we call the set of all these matrices?

   Solution 6. If $M = \begin{bmatrix}a & b\\c & d\end{bmatrix}$, then b must be 0 for $T(M) = 0$. This is the null space.

   c. Describe all matrices that $T(M)$ outputs. What do we call the set of all these matrices?

   Solution 7. If $M = \begin{bmatrix}a & b\\c & d\end{bmatrix}$, then $T(M) = \begin{bmatrix}0 & b\\0 & 0\end{bmatrix}$. This is called the column space.

NOTE: When talking about linear transformations the null space, as we have been calling it, is usually referred to as the kernel and the column space is the range. This will be good to know when reading books that apply the material from this class in your future careers.