

Instructions: This quiz is closed book, closed note, and an individual effort. **Show all work to receive full credit.** Unless the question specifies, you should provide an exact answer. If you get stuck, please attempt to explain what you want to do. This may give more partial credit.

WRITE THIS PARAGRAPH ON WHAT YOU SUBMIT ALONG WITH A SIGNATURE AND DATE.

I, _____, will not under any circumstance use an online source, my peers, my notes, or any other resource besides my own knowledge to complete this quiz. I will show all my work to demonstrate my knowledge on the topic.

1. If A has $\lambda_1 = 2$ with eigenvector $\mathbf{x}_1 = (1, 0)$ and $\lambda_2 = 5$ with eigenvector $\mathbf{x}_2 = (1, 1)$, find A .
2. The block B has eigenvalues 1 and 2. The block C has eigenvalues 3 and 4. The block D has eigenvalues 5 and 7. Find the eigenvalues of A if

$$A = \begin{bmatrix} B & C \\ 0 & D \end{bmatrix} = \begin{bmatrix} 0 & 1 & 3 & 0 \\ -2 & 3 & 0 & 4 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 1 & 6 \end{bmatrix}.$$

Before rolling your sleeves up and doing this out, try to reason what may happen.

3. Diagonalize A and compute $X\Lambda X^{-1}$ to prove this formula for A^k :

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \qquad A^k = \frac{1}{2} \begin{bmatrix} 1 + 3^k & 1 - 3^k \\ 1 - 3^k & 1 + 3^k \end{bmatrix}$$

4. Find two λ 's and \mathbf{x} 's so that $\mathbf{u} = e^{\lambda t}\mathbf{x}$ solves

$$\frac{d\mathbf{u}}{dt} = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} \mathbf{u}.$$

What combination $\mathbf{u} = c_1 e^{\lambda_1 t} \mathbf{x}_1 + c_2 e^{\lambda_2 t} \mathbf{x}_2$ starts from $\mathbf{u}(0) = (5, -2)$?

5. Find an orthonormal matrix Q that diagonalizes $S = \begin{bmatrix} -2 & 6 \\ 6 & 7 \end{bmatrix}$.
6. For which numbers b and c are ALL these matrices positive definite?

$$S = \begin{bmatrix} 1 & b \\ b & 9 \end{bmatrix} \qquad S = \begin{bmatrix} 2 & 4 \\ 4 & c \end{bmatrix} \qquad S = \begin{bmatrix} c & b \\ b & c \end{bmatrix}$$