Instructions: This quiz is closed book, closed note, and an individual effort. **Show all work to receive full credit.** Unless the question specifies, you should provide an exact answer. If you get stuck, please attempt to explain what you want to do. This may give more partial credit.

WRITE THIS PARAGRAPH ON WHAT YOU SUBMIT ALONG WITH A SIGNATURE AND DATE.
I, ______________________, will not under any circumstance use an online source, my peers, my notes, or any other resource besides my own knowledge to complete this quiz. I will show all my work to demonstrate my knowledge on the topic.

1. Find the determinants of U and U^{-1} and U^2.

$$U = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix} \qquad\qquad U = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

Solution 1. $\det(U) = 6$ by upper triangular. $\det(U^{-1} = 1/6$. $\det(U^2) = \det(U)\det(U) = 36$.

$\det(U) = ad$ by upper triangular. $\det(U^{-1}) = 1//(ad)$. $\det(U^2) = \det(U)\det(U) = a^2d^2$.

2. Use Cramer's Rule to solve the following system of equation

$$\begin{aligned} 3x - y &= 1 \\ 5x - 3y &= -5 \end{aligned}$$

Solution 2. $A = \begin{bmatrix} 3 & -1 \\ 5 & -3 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 & -1 \\ -5 & -3 \end{bmatrix}$, and $B_2 = \begin{bmatrix} 3 & 1 \\ 5 & -5 \end{bmatrix}$.

$\det(A) = -4$, $\det(B_1) = -8$, and $\det(B_2) = -20$. Thus, $x = -8/-4 = 2$ and $y = -20/-4 = 5$. Check and it works!

3. Use the cofactor inverse formula to find the inverse of $A = \begin{bmatrix} -3 & 1 & 2 \\ -2 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$.

Solution 3. $C_{11} = -2$, $C_{12} = -4$, $C_{13} = 0$, $C_{21} = 2$, $C_{22} = 6$, $C_{23} = 0$, $C_{31} = -2$, $C_{32} = -4$, $C_{33} = -1$. Thus, $C = \begin{bmatrix} -2 & -4 & 0 \\ 2 & 6 & 0 \\ -2 & -4 & -1 \end{bmatrix} \Rightarrow C^T = \begin{bmatrix} -2 & 2 & -2 \\ -4 & 6 & -4 \\ 0 & 0 & -1 \end{bmatrix}$. $\det(A) = -2 \begin{vmatrix} -3 & 1 \\ -2 & 1 \end{vmatrix} = 2$.

Thus, $A^{-1} = \begin{bmatrix} -1 & 1 & -1 \\ -2 & 3 & -2 \\ 0 & 0 & -1/2 \end{bmatrix}$. Check $AA^{-1} = I$.

4. A box has edges from $(0,0,0)$ to $(1,1,1)$ and $(2,3,1)$ and $(2,1,4)$. Find its volume (remember volume must always be positive).

   Solution 4. Volume= $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 1 & 4 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 11 - 6 - 4 = 1.$

5. Write the above problem as a simple mathematical expression involving a cross product.

   Solution 5. $((1,1,1) \times (2,3,1)) \cdot (2,1,4)$.