Name: **Quiz 1**

Instructions: This quiz is closed book, closed note, and an individual effort. Electronic devices are not allowed on your person (e.g., no cell phones or calculators). Answer each question. **Show all work to receive full credit.** Unless the question specifies, you should provide an exact answer. If you get stuck, please attempt to explain what you want to do. This may give more partial credit.

WRITE THIS PARAGRAPH ON WHAT YOU SUBMIT ALONG WITH A SIGNATURE AND DATE.

I, ____________________, will not under any circumstance use an online source, my peers, my notes, or any other resource besides my own knowledge to complete this quiz. I will show all my work to demonstrate my knowledge on the topic.

1. For each of the below determine whether the statement is true or false. If the statement is false, explain why.

   a. The length of the sum of two vectors is the same as the sum of their lengths.

   b. If we have two vectors $\mathbf{v}_1$, $\mathbf{v}_2 \in \mathbf{R}^3$, then $\mathbf{v}_1 \cdot \mathbf{v}_2 \in \mathbf{R}^3$.

   c. $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \\ 1/5 \end{bmatrix}$ is a unit vector.

   d. $k\mathbf{v} + l\mathbf{u} + e\mathbf{w}$ will always cover all of $\mathbf{R}^3$ for any fixed vectors $\mathbf{v}$, $\mathbf{u}$, and $\mathbf{w}$ with variable $k, l, e \in \mathbf{R}$.

   e. A square matrix will always have a unique inverse.

2. Let $\mathbf{v} = (x, x, y, y)$ have length $2\sqrt{2}$ and $\mathbf{w} = [x, x, x, x]$ have length 4. Determine all possible vectors, $\mathbf{v}$, for which this is true.

3. Consider the following vectors:

$$\mathbf{v} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 5 \\ 2 \\ -4 \end{bmatrix}, \text{ and } \mathbf{u} = \begin{bmatrix} -1 \\ 3 \\ 7 \end{bmatrix}$$

with $k = -1$, $l = 2$, $e = -4$. Answer/find the following:

a. Determine if any of the vectors are perpendicular/orthogonal.

b. $(\mathbf{v} \cdot \mathbf{u})\mathbf{v} + l\mathbf{w}$

c. Find unit vectors in the same direction as each vector.

d. $e\mathbf{v} + k\mathbf{w} + l\mathbf{u}$

4. Given the following system with arbitrary $\mathbf{b}$, find $\mathbf{x}$ with entries in terms of the entries in $\mathbf{b}$.

$$A\mathbf{x} = \begin{bmatrix} 1 & -3 & 1 \\ 0 & -2 & 6 \\ 0 & -15 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \mathbf{b}$$

5. Given your answer above, if $a = 1$, $b = 2$, and $c = -3$ what is $\mathbf{x}$?

6. Below are two matrices A and B, find the following (if not possible, say so and explain):

a. A^{-1}

b. B^{-1}

c. $(AB)^{-1}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -2 & 4 \\ 2 & -2 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

7. Give an example of a square matrix which does not have an inverse. Then show that no inverse exists using elimination. Explain why this shows there is no inverse.