Instructions: This worksheet is for practice and is NOT to be turned in for a grade. Answer each question.

1 Chapter 8

1. Which of these transformations is not linear? The input is $\mathbf{v} = (v_1, v_2)$:
   (a) $T(\mathbf{v}) = (v_2, v_1)$
   (b) $T(\mathbf{v}) = (v_1, v_1)$
   (c) $T(\mathbf{v}) = (0, v_1)$
   (d) $T(\mathbf{v}) = (0, 1)$
   (e) $T(\mathbf{v}) = v_1 - v_2$
   (f) $T(\mathbf{v}) = v_1 v_2$
2. Suppose a linear transformation T satisfies $T(1,1) = (2,2)$ and $T(2,0) = (0,0)$. Find $T(\mathbf{v})$ when:
   (a) $\mathbf{v} = (2,2)$
   (b) $\mathbf{v} = (3,1)$
   (c) $\mathbf{v} = (-1,1)$
   (d) $\mathbf{v} = (a,b)$
3. Suppose T is a linear transformation from V to W. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for V and $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ be a basis for W. Suppose $T(\mathbf{v}_1) = \mathbf{w}_2$ and $T(\mathbf{v}_2) = T(\mathbf{v}_3) = \mathbf{w}_1 + \mathbf{w}_3$. Find the matrix A that corresponds to T. Use A to find $T(\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3)$.
4. Let T be linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$. Take the standard basis for $\mathbb{R}^3$ and let $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ be the basis for $\mathbb{R}^2$. Find the matrix representation A under these bases for the linear transformation T where $T(1,0,0) = (1,0)$ and $T(0,1,0) = (2,2)$ and $T(0,0,1) = (-1,1)$. Use the matrix A to compute $T(1,2,1)$.
5. Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ so that $T\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \right) = \begin{bmatrix} 5v_1 + v_2 \\ -2v_1 + 2v_2 \end{bmatrix}$. Find bases for $\mathbb{R}^2$ so that the matrix representation of T is diagonal.