Instructions: This homework is an individual effort. Answer each question. This is due on **Monday, June 15th**. **Show all work to receive full credit.**

1 Chapter 6

1. Find the eigenvalues and eigenvectors of these four matrices:

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

2. If A has $\lambda_1 = 4$ and $\lambda_2 = 5$ then $\det(A - \lambda I) = (\lambda - 4)(\lambda - 5) = \lambda^2 - 9\lambda + 20$. Find three matrices that have trace $a + d = 9$ and determinant 20 and $\lambda = 4, 5$.

3. Factor these two matrices into $A = X\Lambda X^{-1}$:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$$

4. Write down the most general matrix that has eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. (Hint: Use $X\Lambda X^{-1}$)

5. The rabbit population shows fast growth (from 6r) but loss to wolves (from -2w). The wolf population always grows in this model:

$$\frac{dr}{dt} = 6r - 2w \qquad \text{and} \qquad \frac{dw}{dt} = 2r + w.$$

   Find the eigenvalues and eigenvectors. If $r(0) = w(0) = 30$ what are the populations at time t? After a long time, what is the ratio of rabbits to wolves?

6. Find the eigenvalues and the unit eigenvectors of $S = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$.(Hint: Use the fact that S is symmetric.)

7. Find an orthonormal matrix Q that diagonalizes $S = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{bmatrix}$.

8. Which of S_1, S_2, S_3, S_4 has positive eigenvalues? Use a test, don't compute the $\lambda' s$.

$$S_1 = \begin{bmatrix} 5 & 6 \\ 6 & 7 \end{bmatrix} \quad S_2 = \begin{bmatrix} -1 & -2 \\ -2 & -5 \end{bmatrix} \quad S_3 = \begin{bmatrix} 1 & 10 \\ 10 & 100 \end{bmatrix} \quad S_4 = \begin{bmatrix} 1 & 10 \\ 10 & 101 \end{bmatrix}$$

9. Compute the upper left determinants of S to establish positive definiteness. Verify that their ratios give the second and third pivots.

$$S = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 3 \\ 0 & 3 & 8 \end{bmatrix}$$