Name: **Homework 2**

Instructions: This homework is an individual effort. Answer each question. This is due on **Monday, June 8th**. **Show all work to receive full credit.**

1 Chapter 5

1. If a 4×4 matrix has determinant $\det(A) = \frac{1}{2}$, find $\det(2A), \det(-A), \det(A^2)$ and $\det(A^{-1})$.

2. True or false, with reason if true or a counterexample if false:

   a. The determinant of $I + A$ is $1 + \det(A)$.

   b. The determinant of ABC is $|A||B||C|$.

   c. The determinant of $4A$ is $4|A|$.

3. Reduce A to an upper triangular matrix and find $\det(A)$.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

4. Compute the determinants of A and B. Are their columns independent?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

5. The n x n determinant C_n has $1's$ above and below the main diagonal: $C_1 = |0|$, $C_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$,

$C_3 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$, $C_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$

   a. What are these determinants C_1, C_2, C_3, C_4?

   b. By cofactors find the relation between C_n and C_{n-1} and C_{n-2}. Find C_{10}.

6. Solve these linear equations by Cramer's Rule:

a.

$$\begin{aligned} 2x + 5y &= 1 \\ x + 4y &= 2 \end{aligned}$$

b.

$$\begin{aligned} 2x + y &= 1 \\ x + 2y + z &= 0 \\ y + 2z &= 0 \end{aligned}$$

7. Find A^{-1} form the cofactor formula $C^T / \det(A)$.

a. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 7 & 1 \end{bmatrix}$ b. $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

8. Find the cofactors of A and multiply AC^T to find $\det(A)$:

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 6 & -3 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

If you change that 4 to 100, why is $\det(A)$ unchanged?

9. a. Find the area of the parallelogram with edges $\mathbf{v} = (3, 2)$ and $\mathbf{w} = (1, 4)$.

b. Find the area of the triangle with sides $\mathbf{v}, \mathbf{w}$, and $\mathbf{v} + \mathbf{w}$. Draw it.

10. A box has edges from $(0, 0, 0)$ to $(3, 1, 1)$ and $(1, 3, 1)$ and $(1, 1, 3)$. Find its volume.

11. Find $\mathbf{u} \times \mathbf{v}$ when $\mathbf{u} = (1, 1, 1)$ and $\mathbf{v} = (5, -1, 7)$.