Instructions: This homework is an individual effort. Answer each question. This is due on **Monday, June 1st**. **Show all work to receive full credit.**

1 Rest of Chapter 3

1. Find a basis and the dimension for each of the four subspaces associated with the following matrices.

   a. $A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$

   b. $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 8 \end{bmatrix}$

2. Let $V = \left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right)$.

   a. Find a matrix A that has V as its row space .

   b. Find a matrix B that has V as its nullspace.

   c. Find AB.

2 Chapter 4

1. If $\mathbf{P}$ is the plane of vectors in $\mathbb{R}^4$ satisfying $x_1 + x_2 + x_3 + x_4 = 0$, write a basis for $\mathbf{P}^{\perp}$ (The orthogonal complement of P). Construct a matrix with $\mathbf{P}$ as its nullspace.

2. Suppose A is the 4 x 4 identity matrix without its last column. Project $\mathbf{b} = (1, 2, 3, 4)$ onto the column space of A. What is the projection matrix, P?

3. What is the orthogonal complement of $S = span \left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right)$?

4. Project $\mathbf{b}$ onto the line throught $\mathbf{a}$. Check that $\mathbf{e}$ (the error) is perpendicular to $\mathbf{a}$.

   a. $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ and $\mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

   b. $\mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ and $\mathbf{a} = \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}$.

5. In both of the above, find the projection matrix P and find the project $\mathbf{p}$.

6. Project $\mathbf{b}$ onto the column space of A by solving $A^T A\widehat{\mathbf{x}} = A^T\mathbf{b}$ and $\mathbf{p} = A\widehat{\mathbf{x}}$.

   $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$

7. What linear combination of (1,2,-1) and (1,0,1) is closest to $\mathbf{b} = (2, 1, 1)$?

8. Find the line of best fit for the points (0,1), (1,5), (3,13), (4,17). Do any of these points lie on the line?

9. Find the closest parabola to the points (0,1), (1,5), (3,13), (4,17). Do any of these points lie on the parabola?

10. Find the closest cubic to the points (0,1), (1,5), (3,13), (4,17). Do any of these points lie on the cubic?

11. Find an orthonormal basis for the column space of A given:

    $A = \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -4 \\ -3 \\ 3 \\ 0 \end{bmatrix}$.

12. Find the projection of $\mathbf{b}$ onto the column space above.