Instructions: This homework is an individual effort. Answer each question. This is due on **Monday, May 25th**. **Show all work to receive full credit.**

1 Rest of Chapter 2

1. Solve the system $L\mathbf{c} = \mathbf{b}$ to find $\mathbf{c}$. Then solve $U\mathbf{x} = \mathbf{c}$ to find $\mathbf{x}$.

   a. $L = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$.

   b. $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$.

2. Let P be the matrix with 1's on the antidiagonal (i.e. from the top right position to the bottom left position). Describe PAP.

3. Describe in words what the 5 x 5 permutation matrices would do to the matrix A.

   a. $P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

   b. $P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

   c. $P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

4. If $A = A^T$ and $B = B^T$, which are symmetric? Show work as to why or why not.

   a. $A^2 - B^2$

   b. $(A + B)(A - B)$

   c. $ABAB$

2 Chapter 3

1. Construct a 3 x 3 matrix whose column space contains (1,1,0) and (1,0,1) but not (1,1,1). Then construct a 3 x 3 matrix whose column space is a line in $\mathbb{R}^3$.

2. If $S = C(A)$ and $T = C(B)$, then describe the matrix whose column space is $S + T$.

3. Construct a matrix whose column space contains (1,1,1) and whose nullspace is the multiples of (1,1,1,1).

4. What are the special solutions to $A\mathbf{x} = \mathbf{0}$ for the following matrices:

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5. Find the complete solution $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_n$ to $A\mathbf{x} = \mathbf{b}$:

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 4 & 8 \\ 4 & 8 & 6 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 10 \end{bmatrix}$$

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6. Show that (b_1, b_2, b_3) is in the column space if $b_3 - 2b_2 + 4b_1 = 0$.

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 3 & 8 & 2 \\ 2 & 4 & 0 \end{bmatrix}$$

7. Find the rank of the A and A^T depending on q.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & q \end{bmatrix}$$

8. Find the largest possible number of independent vectors among:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \mathbf{v}_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \mathbf{v}_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

9. Find 3 different bases for the column space of $U = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$.