Instructions: This exam is closed book, closed note, and an individual effort. Electronic devices are not allowed on your person (e.g., no cell phones or calculators). Remove any smartwatches and non-religious head-wear. Cheating of any kind will not be tolerated and will result in a grade of zero. Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. Write your answers on the test. You have **24 hours** to finish the exam. Answer all questions to the best of your ability. Unless otherwise specified, you are required to **SHOW ALL YOUR WORK** to receive full credit. The exam has 210 possible points. You will be graded out of 200 points.

WRITE THIS PARAGRAPH ON WHAT YOU SUBMIT ALONG WITH A SIGNATURE AND DATE.

I, ______________________, will not under any circumstance use an online source, my peers, my notes, or any other resource besides my own knowledge to complete this exam. I will show all my work to demonstrate my knowledge on the topic. If I do break this honor code, I will accept a 0 on this assignment.

Questions	Possible	Score		Possible	Score
Question 1	30		Question 6	20	
Question 2	20		Question 7	25	
Question 3	25		Question 8	20	
Question 4	20		Question 9	25	
Question 5	25		Total		

1. We will solve the following system of equations in three ways.

$$\begin{aligned} x + 3y + 2z &= 3 \\ 2x + 7y &= 1 \\ x + 3y + 3z &= 4 \end{aligned}$$

   a. Solve the system of linear equations by use of an augmented matrix.

   b. Solve the system using Cramer's Rule.

   c. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 7 & 0 \\ 1 & 3 & 3 \end{bmatrix}$ and use it to solve the system (Hint: We are solving $A\mathbf{x} = \mathbf{b}$. How can we achieve a formula with the inverse?).

2. Find a **unit** vector that is orthogonal to both $\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ -1 \\ 3 \end{bmatrix}$.

3. Find the complete solution to the equation

$$\begin{bmatrix} 2 & 1 & -1 & 3 \\ 6 & 3 & -3 & 10 \\ -4 & -2 & 2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 23 \\ 18 \end{bmatrix}.$$

4. Find the line of best fit for the points $(-1, 2), (0, -2), (1, 10), (2, 8)$.

5. Consider the linear transformation T from $\mathbf{R}^2$ to $\mathbf{R}^2$ given by $T = \left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \right) = \begin{bmatrix} 4v_1 + 2v_2 \\ 2v_1 + 7v_2 \end{bmatrix}$.

   a. Find the matrix representation A corresponding to T under the standard basis for $\mathbf{R}^2$, (1,0) and (0,1).

   b. Find the eigenvalues and eigenvectors for A found in part (a).

   c. Now, let $\{\mathbf{x}_1, \mathbf{x}_2\}$ be the basis for $\mathbf{R}^2$, where $\mathbf{x}_1$ and $\mathbf{x}_2$ are eigenvectors of A. Find the matrix representation B corresponding to T under this new basis.

6. One of each of the following matrices matches each of the cases below. Match the matrices with the cases and **justify**. No points will be given for just matching them, even if the answer is correct.

   Let r be the rank of the matrix, m be the number of rows, and n be the number of columns.

   1 $r = m = n$.

   2 $r = m < n$.

   3 $n = r < m$.

   4 $r < m$ and $r < n$.

   a.
   $$M = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ -2 & -3 \end{bmatrix}$$

   b.
   $$A = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 1 & 3 & 1 & 6 \end{bmatrix}$$

   c.
   $$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix}$$

   d.
   $$H = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 0 \\ 2 & 5 & 9 \end{bmatrix}$$

7. The following questions pertain to factoring the matrix A so that $PA = LU$, where

   $$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}.$$

   a. What matrix P will make $PA = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 0 & 1 \\ 3 & 4 & 5 \end{bmatrix}$?

   b. What elimination matrices do we need to left multiply PA by to get the product to be upper triangular? What is U?

   c. What will be the matrix L?

8. Find an orthonormal basis for the vector space spanned by the columns of the matrix below.

$$A = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

9. Diagonalize the matrix A below via eigenvalues and find a formula for A^k for any integer k.

$$A = \begin{bmatrix} 5 & 1 \\ 0 & 4 \end{bmatrix}.$$