Name: **Exam 2**

Instructions: This exam is closed book, closed note, and an individual effort. Electronic devices are not allowed on your person (e.g., no cell phones or calculators). Remove any smartwatches and non-religious head-wear. Cheating of any kind will not be tolerated and will result in a grade of zero. Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. Write your answers on the test. You have **24 houra** to finish the exam. Answer all questions to the best of your ability. Unless otherwise specified, you are required to **SHOW ALL YOUR WORK** to receive full credit. The exam has 110 possible points. You will be graded out of 100 points.

WRITE THIS PARAGRAPH ON WHAT YOU SUBMIT ALONG WITH A SIGNATURE AND DATE.

I, ______________________, will not under any circumstance use an online source, my peers, my notes, or any other resource besides my own knowledge to complete this exam. I will show all my work to demonstrate my knowledge on the topic. If I do break this honor code, I will accept a 0 on this assignment.

Questions	Possible	Score		Possible	Score
Quiestion 1	20		Question 5	10	
Question 2	10		Question 6	20	
Question 3	10		Question 7	10	
Question 4	20		Question 8	10	
			Total		

True/False

1. Instructions: You must determine if the statement is true or false. If you say false, provide a counterexample. If you say true, explain your reasoning.

   a. Let A be a 3×3 matrix with $\det(A) = 1$. Then $\det(2A) = 2$.

   b. For every 2×2 matrices A and B, we have that $\det(A + B) = \det(A) + \det(B)$.

   c. Performing row operations on a matrix does not change the eigenvalues.

   d. Every positive-definite matrix is invertible.

   e. If S is a positive-definite matrix, then S^2 is also a positive-definite matrix.

Open-ended

2. Consider the predator vs prey model where the gazelle population shows fast growth (from 5g) but loss to lions (from -2ℓ), while the lion population always grows:

$$\frac{dg}{dt} = 5g - 2\ell \qquad \text{and} \qquad \frac{d\ell}{dt} = g + 2\ell.$$

Solve the differential equation

$$\frac{d\mathbf{u}}{dt} = \begin{bmatrix} 5 & -2 \\ 1 & 2 \end{bmatrix} \mathbf{u}(t), \qquad \text{where } \mathbf{u}(t) = \begin{bmatrix} g(t) \\ \ell(t) \end{bmatrix}.$$

Leave two constants in your solution.

3. Use **Cramer's Rule** to solve for the unknown coefficients above when the initial populations are $g(0) = 50$ and $\ell(0) = 30$ for the above.

4. Consider the matrix $S = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$.

   a. Determine if S is positive-definite using one of the four tests.

   b. Find an orthonormal matrix Q which diagonalizes S.

   c. Using the diagonalization in (b), find a formula for S^k.

   d. Use the formula in (c) to calculate S^4.

5. a. Given that the eigenvalues of a 2×2 matrix are 2 and 3, what is the determinant of that matrix?

   b. Suppose the trace of a 2×2 matrix is 7 and one of the eigenvalues is $\lambda_1 = -3$, what is the other eigenvalue λ_2?

   c. Suppose the trace of a 2×2 matrix is 4 and the determinant is 3, what are the eigenvalues of the matrix?

   d. Suppose the trace of a 3×3 matrix is 6, the determinant is 6, and one of the eigenvalues is $\lambda_1 = 1$, what are the other two eigenvalues of the matrix?

6. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix}$ using the cofactor inverse formula.

(Hint: Show OR EXPLAIN how you got the cofactors)

7. Find a 2×2 matrix with eigenvalues $\lambda_1 = 3$ and $\lambda_2 = -2$ and corresponding eigenvectors $\mathbf{x}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

8. Let $\mathbf{u} = (1, 2, 1)$ and $\mathbf{v} = (3, 7, 1)$.

   a. Find $\mathbf{u} \times \mathbf{v}$.

   b. Let $\mathbf{w} = (5, -2, 3)$. Find the triple product of the three vectors.

   c. Find the area of the triangle spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.