Name: **Exam 1**

Instructions: This exam is closed book, closed note, and an individual effort. Electronic devices are not allowed on your person (e.g., no cell phones or calculators). Remove any smartwatches and non-religious head-wear. Cheating of any kind will not be tolerated and will result in a grade of zero. Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. Write your answers on the test. You have **24 houra** to finish the exam. Answer all questions to the best of your ability. Unless otherwise specified, you are required to **SHOW ALL YOUR WORK** to receive full credit. The exam has 110 possible points. You will be graded out of 100 points.

WRITE THIS PARAGRAPH ON WHAT YOU SUBMIT ALONG WITH A SIGNATURE AND DATE.

I, _____, will not under any circumstance use an online source, my peers, my notes, or any other resource besides my own knowledge to complete this exam. I will show all my work to demonstrate my knowledge on the topic. If I do break this honor code, I will accept a 0 on this assignment.

Questions	Possible	Score		Possible	Score
MC	20		Question 4	10	
Question 1	10		Question 5	10	
Question 2	20		Question 6	10	
Question 3	20		Question 7	10	
Extra Credit			Total		

Multiple Choice

Each question is worth 1 point!

1. When A, B, C are symmetric, then the transpose of ABC is CBA.
   (a) Duh! True.
   (b) No way! False
   (c) Where's the rest of the alphabet?
2. If $AB = AC$ then $B = C$.
   (a) Are sloths good swimmers? True
   (b) Are elephants just big, hairless, wrinkly dogs? False
   (c) Fun fact: The mantis shrimp has the world's fastest punch.
3. Let $\mathbf{u} = (1, 2, -2)$ and $\mathbf{v} = (2, 1, 2)$.
   (a) $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.
   (b) $\mathbf{u}$ and $\mathbf{v}$ are orthonormal.
   (c) $\mathbf{u}$ and $\mathbf{v}$ are of equal length.
   (d) Both a and c are true.
   (e) None of the above.
4. $\begin{bmatrix} 3 \\ 9 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}\right)$.
   (a) Yessir! True
   (b) No way! False
   (c) Did you spell spam wrong?
5. The matrix $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 3 & 3 & 0 \end{bmatrix}$ is invertible.
   (a) If that is not true, I don't know what is! True
   (b) That matrix hurts me! False
   (c) What's a matrix?

6. The subspaces $V = \text{span}\left(\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}\right)$ and $W = \text{span}\left(\begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}\right)$ are orthogonal.

   (a) By thorough investigation, true!

   (b) By thorough investigation, false!

   (c) By lack of investigation this case is still open.

7. If the columns of a matrix are dependent, then so are the rows.

   (a) Truer than true

   (b) You think you can trick me Tommy? False

   (c) Is this the Krusty Krab?

8. A square matrix has no free variables.

   (a) By math, true.

   (b) Something is fishy... False

   (c) No, this is Patrick.

9. Any 6 vectors in $\mathbf{R}^5$ are linearly dependent.

   (a) YES YES YES YES YES True

   (b) NO NO NO NO NO False

   (c) IDK IDK IDK IDK IDK

10. A linear system of 3 variables has exactly 3 solutions.

    (a) why not? True

    (b) But why though? False

    (c) 21

11. Three vectors in $\mathbf{R}^3$ will always span all of $\mathbf{R}^3$.

    (a) YAS True

    (b) UHHHHH

12. $||\mathbf{u}|| * (\mathbf{u} \cdot \mathbf{v})$ where $\mathbf{u} = (1, 2, 2)$ and $\mathbf{v} = (0, 3, -4)$.

    (a) -18

    (b) -6

    (c) 6

    (d) 18

    (e) 12

13. Let $\mathbf{u} = (\sqrt{2}/2, \sqrt{2}/2, 0, 0)$ and $\mathbf{v} = (0, 0, \sqrt{3}/2, 1/2)$.

    (a) They are both unit vectors only.

    (b) They are orthogonal only.

    (c) They are not orthogonal or unit vectors.

    (d) They are orthonormal.

    (e) None of the above.

14. How many 5 x 5 permutation matrices are there?

    (a) 20

    (b) 120

    (c) 5

    (d) 50

    (e) 60

15. $(AB)^{-1}(C+D)^T(C^T+D^T)^{-1}(AB) =$

    (a) A^TB^TAB

    (b) $C+D$

    (c) AB

    (d) $ABC+ABD$

    (e) Identity

16. $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = LU$ is a factorization where L is:

    (a) $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

    (b) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

    (c) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

    (d) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

    (e) Identity

17. If A is a matrix, then

    (a) $\dim(C(A)) + \dim(N(A))$ is the number of rows.

    (b) $\dim(N(A)) + \dim(C(A))$ is the number of columns.

    (c) $\dim(N(A)) + \dim(C(A))$ is the rank.

    (d) $\dim(N(A)) + \dim(C(A))$ is the sum of the number of columns and rank.

    (e) $\dim(N(A)) + \dim(C(A))$ is the sum of the number of rows and the rank.

18. The null space is just the zero vector exactly when the columns are independent vectors.

    (a) Truth has been spoken

    (b) Lying is a bad habit! False

    (c) The null space is always empty no matter what!

19. The left null space is the orthogonal complement of

    (a) $N(A)$

    (b) $C(A)$

    (c) $C(A^T)$

    (d) $N(A^T)$

    (e) None of the above.

20. If $\mathbf{p}$ is the projection of $\mathbf{b}$ on to the line $\mathbf{a}$, then $||\mathbf{p}|| = ||\mathbf{a}||$.

    (a) I got this one for sure! True

    (b) Uhhh I don't think so Tommy. False

    (c) Are we done yet?

Open-ended

1. Determine the angle between the vectors $\mathbf{v} = (3, 2, 6)$ and $\mathbf{u} = (3, -1 - 3\sqrt{3}, -3 + \sqrt{3})$. Leave your answer in terms of $\cos^{-1}$.

2. Consider the following linear system of equations:

$$\begin{aligned} x + 2y - 3z &= -1 \\ 2x + 3y - 9z &= -7 \\ 3x + 7y - 7z &= 0 \end{aligned}$$

   a. Solve the system.

   b. Use the process above to determine if $\begin{bmatrix} -1 \\ -7 \\ 0 \end{bmatrix}$ is in span $\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} -3 \\ -9 \\ -7 \end{bmatrix} \right)$. Explain.

   c. Let A be the matrix you form when writing the system as $A\mathbf{x} = \mathbf{b}$. Find the LU factorization of A(you may cite previous work to explain your process).

   d. Find A^{-1} if it exists. If it does not exist, explain why.

3. Consider the linear system:

$$\begin{aligned} x + 4y + 2z + 3w &= -3 \\ 2x + 8y + z &= -3 \\ -3x - 12y - 10z - 17w &= 13 \end{aligned}$$

   a. Write the system in the form $A\mathbf{x} = \mathbf{b}$. Then find the span of each of the four subspaces of A.

   b. Find the complete solution to the system, $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_n$.

4. Create an orthonormal basis for $\mathbf{R}^2$ with the basis $\left\{ \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.

5. Let (-1,4), (0,-4), (1,10), (2,6) be a set of points we want to fit to a parabola.

   a. Write a linear system in the following form to attempt to answer this question: $A\mathbf{x} = \mathbf{b}$.

   b. Use the above to determine the parabola of best fit to the 4 points.

6. Determine if the vectors $\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 17 \\ -60 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ are linearly independent or linearly dependent.

7. For each of the below, determine a system of linear equations for which the property is holds.

   a. No solutions.

   b. The only solution is $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$.

   c. The only solutions are vectors of the form $\begin{bmatrix} -a \\ 2a \end{bmatrix}$.

8. Extra Credit: Suppose that the 3 x 4 matrix A has the vector $\mathbf{s} = (2, 3, 1, 0)$ as the only special solution to $A\mathbf{x} = \mathbf{0}$. What is the exact row reduced echelon form of A?