Applications of the Complete Solution to $Ax = b$

Tommy Luckner
Department of Mathematics

Overview

We will interpret the particular and special solution to $Ax = b$ and see an application of this.

Interpretation

The complete solution to $Ax = b$ is $x = x_p + x_s$, where $Ax_p = b$, and $Ax_s = 0$. What does this mean practically? Suppose, for example, we have $x = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix}^T$, with x_1 and x_3 being pivot variables, and x_2 and x_4 being free variables. If we have the following solution

$$x_p = \begin{pmatrix} 2 \\ 0 \\ -3 \\ 0 \end{pmatrix}, \qquad x_s = \begin{pmatrix} 4 & -5 \\ 1 & 0 \\ 1 & 6 \\ 0 & 1 \end{pmatrix},$$

then

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -3 \\ 0 \end{pmatrix} + x_2 \cdot \begin{pmatrix} 4 \\ 1 \\ 1 \\ 0 \end{pmatrix} + x_4 \cdot \begin{pmatrix} -5 \\ 0 \\ 6 \\ 1 \end{pmatrix}.$$

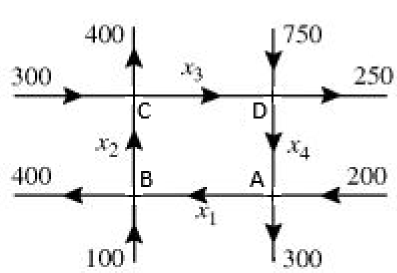
Writing this individually, we have

$$\begin{aligned} x_1 &= 2 + 4x_2 - 5x_4 \\ x_2 &= x_2 \\ x_3 &= -3 + 1x_2 + 6x_4 \\ x_4 &= x_4. \end{aligned}$$

x_2 and x_4 can be any value, and x_1 and x_3 depend on them.

In-Class Exercise

Consider traffic moving along the one way streets as in the following diagram.



The direction of the traffic is indicated by the arrows. We have measurements for the traffic flow, in vehicles per hour (vph), for certain segments, as indicated by the numbers. The traffic flow for some segments are unknown, as indicated by the variables.

What is the minimum traffic flow along each unknown segment?

Answers

The minimum traffic flow along each segment is as follows.

$$\begin{array}{lrrl} AB & \text{min}: & 400 & vph \\ BC & \text{min}: & 100 & vph \\ CD & \text{min}: & 0 & vph \\ DA & \text{min}: & 500 & vph \end{array}$$