LU Factorization

Tommy Luckner
Department of Mathematics

Overview

We will use MATLAB to find the LU factorization of a square matrix.

Commands Needed

Given that we have an $m \times n$ matrix A in memory,

- `A(i,j)` retrieves element (i, j) of A.
- `A(i,1:n)` retrieves row i of A.
- `A(1:m,j)` retrieves column j of A.
- `A(2:4,3:7)`, for example (assuming $4 \leq m$ and $7 \leq n$), retrieves a submatrix of A from rows 2 through 4 and columns 3 through 7.

Activities

1. In the command window, define the matrix

$$A = \begin{pmatrix} 8 & -3 & 6 & 1 \\ 8 & 7 & 4 & 8 \\ 4 & 4 & 5 & 9 \\ -4 & -1 & 2 & 2 \end{pmatrix}.$$

2. To initialize L and U, type

```
>> L = eye(4)      % L is the 4 by 4 identity matrix initially
>> U = A           % U is A initially
```

3. To perform elimination on the first column, type

```
>> L(2,1) = U(2,1)/U(1,1);                  % Here we are forcing
>> U(2,1:4) = U(2,1:4)-L(2,1)*U(1,1:4);    % U(2,1), U(3,1), and U(4,1)
>> L(3,1) = U(3,1)/U(1,1);                  % to be 0. Why are we
>> U(3,1:4) = U(3,1:4)-L(3,1)*U(1,1:4);    % performing valid
>> L(4,1) = U(4,1)/U(1,1);                  % computations?
>> U(4,1:4) = U(4,1:4)-L(4,1)*U(1,1:4);
```

Can we use A to form L?

4. To perform elimination on the second column, type

```
>> L(3,2) = U(3,2)/U(2,2);                 % Here we are forcing
>> U(3,2:4) = U(3,2:4)-L(3,2)*U(2,2:4); % U(3,2) and U(4,2)
>> L(4,2) = U(4,2)/U(2,2);                 % to be 0.
>> U(4,2:4) = U(4,2:4)-L(4,2)*U(2,2:4);
```

   Note here that we do not need the entries in the first column (or second row). Why?

5. To perform elimination on the third column, type

```
>> L(4,3) = U(4,3)/U(3,3);
>> U(4,3:4) = U(4,3:4)-L(4,3)*U(3,3:4);
```

   Note here that we do not need the entries in the first two columns (or second two rows). Why? Why do we stop at this column?

6. We now have an LU factorization of A. To check if it is correct, type

```
>> A - L*U
```