Thoughts:

This is one of my favorite sections of Calculus 1. Why? Because it allows me to be lazy! You are not always given a function in terms of just $x$. You may have something like: $xy + y = x$ and you want the derivative with respect to $x$. Now what? Well, what we do is consider $y$ a function of $x$ and approach accordingly! In the example I gave, we have a product rule ($xy$ is a product of 2 functions of $x$) and a single function with respect to $x$. One the right we have a single $x$. Thus, we have the following: $y + xy' + y' = 1$. A little confusing, but let me explain. We never do anything we $y$ except make it $y'$ when we are taking its derivative. That is it!
Now this should concern you because you learned next to nothing about the derivative! Let's change that by finding $y'$! Let's solve for it.

$$xy' + y' = 1 - y$$

$$y'(x+1) = 1 - y$$

$$y' = \frac{1-y}{x+1}.$$

This is useful in answering questions where you need the value of the slope of the tangent line given a point (or an $x$ where you find the $y$-coordinate). That's it! However, this is not its only application. If you can tell me how to find the derivative of this function without using a technique I show you, write it up and present it at math conferences because there is no other way that we know of! Here is the function: $y = x^x$. Weird function, but implicit differentiation helps us here if we use ln on both sides first!

$$\ln(y) = x\ln(x).$$

Now we can use implicit differentiation (chain rule on the left and product rule on the right)!

$$\frac{y'}{y} = \ln(x) + 1$$

$$\Rightarrow y' = y(\ln(x) + 1).$$

Tada! There is our derivative with respect to $x$. This is the value of implicit differentiation. Another example is for inverse trig functions! Here: $y = \arccos(x)$. Take cos of both sides. $\cos(y) = x$. Now use implicit! Try it with others!

Problems: Find $y'$ or $\dfrac{dy}{dx}$ for the functions below.

1. $x^2 + 2xy - y^2 + x = 2$
2. $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$
3. $y \sin(2x) = x \cos(2y)$
4. $x^{2/3} + y^{2/3} = 4$
5. $e^{x/y} = x - y$
6. $\sqrt{x+y} = 1 + x^2y^2$
7. $\tan(x - y) = \dfrac{y}{1 + x^2}$
8. $y = \arctan(x)$
9. $y = \arctan(x^2y) = x + xy^2$
10. $y = x^{x^x}$
11. For 1 - 4 use the points (1,2), $(0, 1/2)$, $(\pi/2, \pi/4)$, and $(-3\sqrt{3}, 1)$ respectively to find tangent lines at the points for the curves.