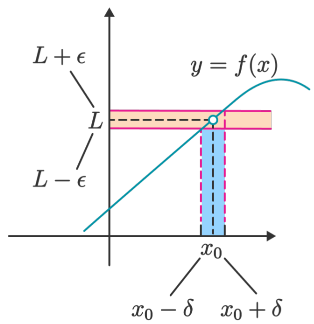
Thoughts:

Epsilon Delta- This is a particularly hard topic for me to explain thoroughly. The best way is via a picture.



What the epsilon delta definition of a limit approaching x_0 is saying in terms of this picture is simply L is our limit if for a chosen epsilon, delta exists where my interval on the x-axis gives values of x where $f(x)$ is within epsilon of L. This probably still makes your head hurt/spin and it probably won't stop. However, let'd make more sense of this with some examples. Let's show this is true by this definition.

$$\lim_{x\to 3} 4x-5 = 7$$

Let's just plop this into our definition.

$$|x-3| < \delta, \qquad |(4x-5)-7| < \epsilon.$$

Simplify and find our left inequality!

$$|(4x-5)-7| = |4x-12| = 4|x-3| < \epsilon \Rightarrow |x-3| < \frac{\epsilon}{4}.$$

If we take $\delta = \frac{\epsilon}{4}$ we have proved this!
Let's try another.

$$\lim_{x\to 3} x^2 = 9$$

Once again plop in.

$$|x-3| < \delta, \qquad |x^2-9| < \epsilon.$$

Let's do some simplifying.

$$|x^2-9| = |(x-3)(x+3)| < \epsilon.$$

Thus, we need to bound the $x+3$! Then we are home free! Since we are looking at x as we approach 3, in particular, x close to 3; let's use a distance of 1 from 3! Therefore, $|x+3| < 7$ for $2 \leq x \leq 4$. Delta with be smaller than this so we are ok! Over this interval, we also have $|x-3| < 1$. Let's see what happens when we use this.

$$|(x-3)(x+3)| < 7|x-3| < \epsilon \Rightarrow |x-3| < \frac{\epsilon}{7}.$$

OR we can just use $|x-3| < 1$. Thus, δ should be the smaller of the 2 of these. Namely, $\delta = \min\{1, \frac{\epsilon}{7}\}$.
THIS IS HARD! Please practice!

Continuity- We discussed this lightly in class, but we know a function is continuous at a if the limit is the same as $f(a)$. This brings up the idea of discontinuity. When we seek continuity we are really looking for an x such that we do not have this condition! Examples include holes, piece-wise functions, 'jumps', and asymptotes. A nice property of continuous functions is that you can move the limit inward for a cont. function. What does this mean? Let's look at the function x^2. This function is continuous for all x. Thus, any limit of this can be represented as the following: $\lim x^2 = (\lim x)^2$. This is possible as long as the function is continuous at the limit point! Try looking at general sets of functions and seeing if they are continuous. For example: polynomials, rational functions, complex functions, trig functions, inverse trig functions, etc. This brings us to the big theorem here; The Intermediate Value Theorem!
If f is continuous on $[a,b]$ and N between $f(a)$ and $f(b)$ not equal, then there is a c in (a,b) such that $f(c) = N$.
We should always ask, so what? An example can tell you why this is valuable.
Show the following has a root.

$$4x^3 - 6x^2 + 3x - 2$$

Something, say c, is a root if $f(c) = 0$. The Intermediate Value Theorem is a picking something bigger and something smaller than 0 means I have a c for which this is true since this polynomial is continuous! To save time and computation, $f(1) = -1$ and $f(2) = 12$. Thus, somewhere between 1 and 2 there is a c for which $f(c) = 0$! Done! Notice we did not find the root, but we know it exists.

Inifinte limits- I talked about these briefly as an exercise in class. I mentioned that the big technique is to divide by the highest power of x in your function for rational functions. There are more techniques such as multiplying by the conjugate and the dividing by largest power with respect to a root. You do need to be careful!
I mentioned lightly in class that dividing by the highest power has a discrepency in math. In this class it seems you will need to divide by the highest power in the **denominator only**. The idea is the following:

$$\lim_{x\to\infty} \frac{x^2}{x} = \lim_{x\to\infty} x = \infty$$

BUT if we do divide by x^2 everywhere we get a 0 denominator which is undefined!
Lastly, if $\lim_{x\to\pm\infty} f(x) = L$ then $y = L$ is a horizontal asymptote. i.e. draw a line here and the graph of the function approaches this line in but does not touch it in the direction of infinity or negative infinity accordingly.

1. Prove the following via epsilon delta:

   a. $\lim_{x \to 1} \frac{2+4x}{3} = 2$

   b. $\lim_{x \to -2} x^2 - 1 = 3$

   c. $\lim_{x \to 2} x^3 = 8$

2. Determine if the function is continuous. If it is not, give all the discontinuous points.

   a. $\begin{cases} x+1 & x \leq 1 \\ \frac{1}{x} & 1 < x < 3 \\ \sqrt{x-3} & x \geq 3 \end{cases}$

   b. $\begin{cases} x+2 & x < 0 \\ e^x & 0 \leq x \leq 1 \\ 2-x & x > 1 \end{cases}$

3. Prove the folllowing have a root on the interval given by IVT.

   a. $\sqrt[3]{x} = 1 - x$, (0,1)

   b. $\sin(x) = x^2 - x$, (1,2)

4. Find the limits or show they do not exist ($\pm\infty = DNE$).

   a. $\lim_{x \to \infty} \frac{(2x^2+1)^2}{(x-1)^2(x^2+x)}$

   b. $\lim_{x \to \infty} \frac{\sqrt{9x^6 - x}}{x^3+1}$

   c. $\lim_{x \to -\infty} x + \sqrt{x^2+2x}$

5. Sketch the graph given the following: $\lim_{x \to 2} f(x) = \infty$, $\lim_{x \to -2^+} f(x) = \infty$, $\lim_{x \to -2^-} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = 0$, $\lim_{x \to \infty} f(x) = 0$, and $f(0) = 0$.