Math 115 Final Review

1 Algebraic Expressions

1. Simplify each expression. Use absolute values if necessary.

   (a) $\sqrt{(-3)^2}$

   (b) $\sqrt{(-x)^2}$

   (c) $(a^4)^{\frac{1}{2}}$

   (d) $\left(\frac{d^6}{25}\right)^{-2}$

(e) $\dfrac{(12-2(-3+5))^3}{5^2-7(5-2)}+7$

2. Find each product, combine any like term.

   (a) $(2x-2)(5x+7)$

   (b) $(x^2+2x+5)(2x-1)$

(c) $(2x^2 - 3x + 4)^2$

3. Factor each polynomial.

(a) $a^2 - 8a + 7$

(b) $4t^2 + 5t - 9$

(c) $27w^4 - 8w$

4. Simplify each expression use absolute values if necessary.

(a) $\sqrt{x^2 - 2x + 1}$

(b) $\dfrac{x^2 + x - 6}{x^2 + 2x + 1} \div \dfrac{x^2 - 4}{x^2 + 3x + 2}$

2 Algebraic Equations and Graphing Basics

1. Solve $|2x - 5| = 6$ for x.

2. Solve $K = 5/9(F - 32) + 273$ for F

3. How many gallons of a 60% antifreeze solution must be mixed with 60 gallons of 20% antifreeze to get a mixture that is 50% antifreeze?

4. Find the equation of the line in point-slope form and slope-intercept form that passes through the points $(-5, -2)$ and $(5, 12)$.

5. Find the equation of the line in slope-intercept form that passes through $(7, -3)$ and perpendicular to the line $y = \frac{1}{2}x + 3$.

6. Find the equation of the line in slope-intercept form that passes through $(1, 2)$ and parallel to the line $y = 2x - 5$.

7. Graph the line $y - 2 = \frac{1}{3}(x + 3)$.

8. Solve the following quadratic equations by factoring if posibble. If not use the quadratic formula to find all real or imaginary solutions.

   (a) $x^2 - 7x = 30$

   (b) $2x^2 - x + 5 = 0$

3 Functions

1. Is $f = \{(2, -1), (3, 4), (1, 0), (2, 5)\}$ a function?

2. Is $f = \{(1, 2), (2, 3), (3, 3), (4, 2)\}$ a function?

3. What test can be used to tell if the graph of a relation is the graph of a function?

4. Determine whether the following equations defines y as a function of x.

   (a) $y = -10x + 2$

   (b) $x = y^6$

   (c) $x = y^{\frac{1}{4}}$

5. Let $f(x) = \sqrt{81 - x^2}$. Sketch the graph and state the domain and range. Identify any intervals on which $f(x)$ is increasing, decreasing, or constant.

6. Let

$$f(x) = \begin{cases} \sqrt{x+6} & \text{for } -6 \leq x \leq 2 \\ x & \text{for } x \geq 2 \end{cases}.$$

Graph the function and determine the domain and range.

7. For each of the following find and simplify the difference quotient.

   (a) $f(x) = 3x^2 - 8x + 7$

   (b) $f(x) = \sqrt{x+2}$

(c) $f(x) = \dfrac{1}{x+1}$

8. For $f(x) = 4x + 3$ and $g(x) = \sqrt{x+1}$ find $f \circ g(x)$ and $g \circ f(x)$.

9. Let $f(x) = |x|, g(x) = x - 3$, and $h(x) = \sqrt{x}$ Write $N(x) = \sqrt{|x| - 3}$ as a composition of f, g, and h.

10. What test, given the graph of a function, can be used to test if that function has an inverse function?

11. For each function determine if the function is one-to-one.

    (a) $f = \{(1,2),(2,3),(3,2),(4,5)\}$

    (b) $f = \{(1,2),(2,5),(3,11),(4,17)\}$

    (c) $f(x) = x^2$

    (d) $f(x) = x^5$

12. Find the inverse function of each of the following functions

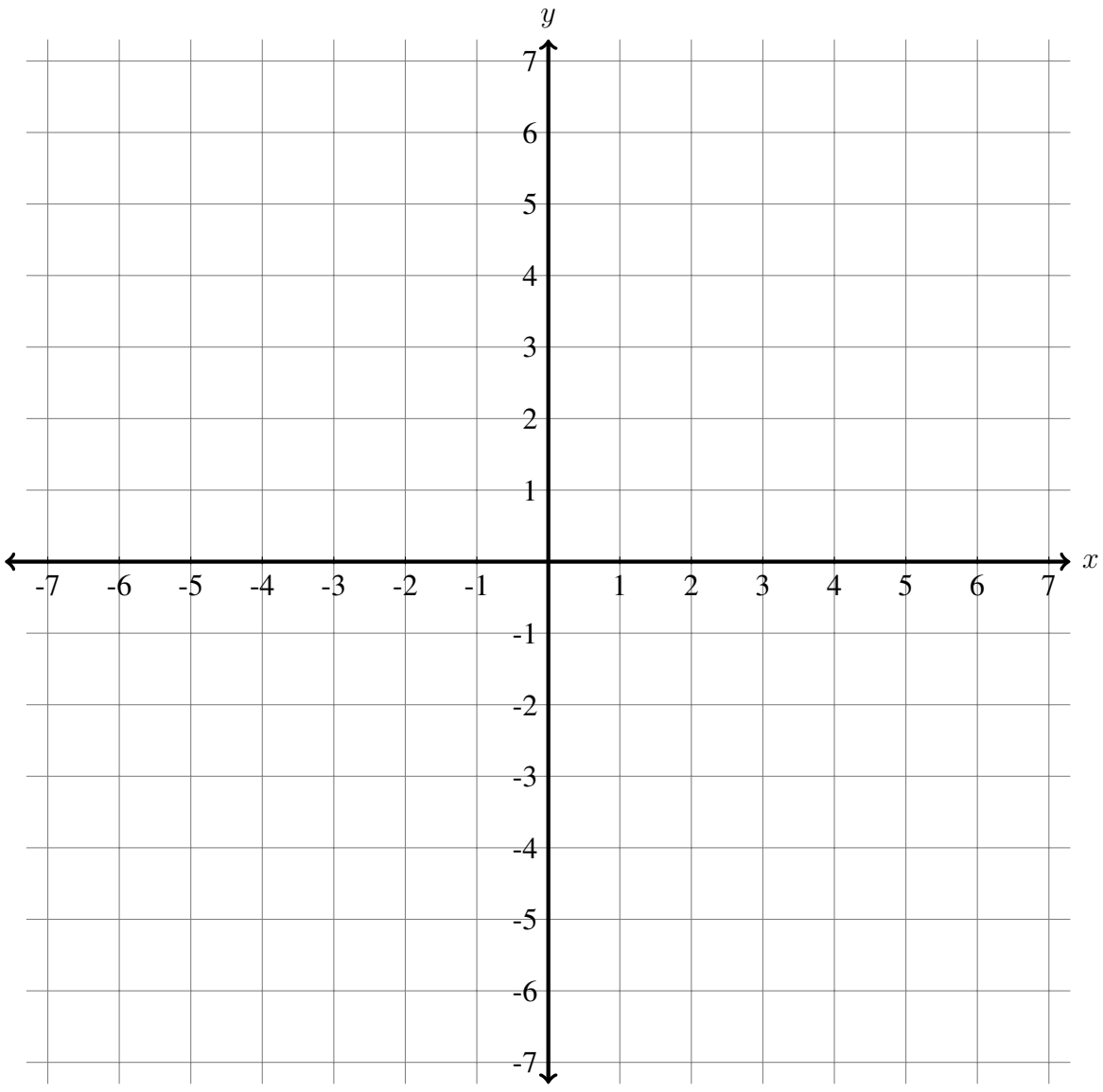
    (a) $f = \{(1,2),(2,3),(3,5),(4,7)\}$

(b) $f(x) = x^3 + 5$.

4 Polynomials

1. Write the quadratic function, $y = x^2 + 4x$, in vertex form($y = a(x - h)^2 + k$) and sketch its graph.(Hint complete the square!)

   Vertex form:



2. Let $P(x) = x^4 - 2x^3 - 2x^2 + 2x + 1$.

   (a) The possible rational roots of $P(x)$ are:

   (b) Find all roots of $P(x)$.

3. How many roots(real or complex) does $x^3 + 29x^2 + 100x + 7$ have?

4. How many roots does a degree n polynomial have?

5. Let $P(x)$ be a polynomial with real coefficients and with $2 - 3i$ as a root. What is one other root of $P(x)$.

6. Find a polynomial in general form with real coefficients that has 4 and $5i$ as roots.

7. Use Decrates' Rule of Signs to find the possibilities for the roots of

$$x^7 + 10x^6 - 100x^5 - 50x^4 + 35x^2 + 40x - 5$$

8. Find all real and imaginary solutions to $x^4 + 6x^2 - 40$.(Simplify your answer, but give an exact answer using radicals as needed. Express complex numbers in terms of i.)

5 Exponential and Logarithmic Functions

1. Solve the following equations for x.

   (a) $10^x = 0.0001$

   (b) $5^x = 125$

   (c) $\log_2(x) = 4$

(d) $\log_3(81) = x$

(e) $\log_x(\frac{1}{27}) = 3$

2. Find the inverse function for each of the following functions.

(a) $f(x) = e^{x+2} - 5$

(b) $f(x) = \log_6(3x - 10) + 3$

3. For each of the following logarithmic expressions use logarithm laws to rewrite each as a single logarithm.

   (a) $2\ln(x) + \frac{1}{2}\ln(y) - 5\ln(z)$

   (b) $5\log_5(x) - \log_5(y) - \frac{1}{3}\log_5(y) + 7\log_5(z)$

4. For each of the following rewrite each logarithmic expression as a sum and/or difference of simple logarithms. Simplify any simple logarithms if possible.

   (a) $\ln\left(\dfrac{x^5\sqrt[3]{y}}{z^2}\right)$

(b) $\log_3 \left(\dfrac{\sqrt{3}(x+y)^5}{z^{\frac{3}{2}}} \right)$

5. Solve the following equations for x

   (a) $e^{2x-3} = 1$

   (b) $\dfrac{1}{27} \cdot 9^{x^2} = 3^{-1}$

(c) $5^{x+2} = 7$

(d) $\ln(x-1) = \ln(x+1) + 2$

(e) $\log_3(x-2) = 1 - \log_3(x+2)$

6 Trigonometric Functions

1. Determine if the given angles, α and β, are coterminal.

   (a) $\alpha = 1000^\circ, \beta = -440^\circ$

   (b) $\alpha = 117\pi/7, \beta = 5\pi/7$

2. Find the **exact** value of each: (by exact I mean if you give me a decimal because you found it using a calculator you will recieve no credit)

   (a) $\sin(-\pi/6) =$

   (b) $\cos(4\pi/3) =$

   (c) $\tan(1001\pi/4) =$

   (d) $\sec(17\pi/3) =$

   (e) $\csc(-300^\circ) =$

   (f) $\cot(-1290^\circ) =$

3. Find the exact value of the other five trigonometric functions, given that $\cos(\alpha) = \frac{8}{17}$ and α is in quadrant I.

   (a) $\sin(\alpha) =$

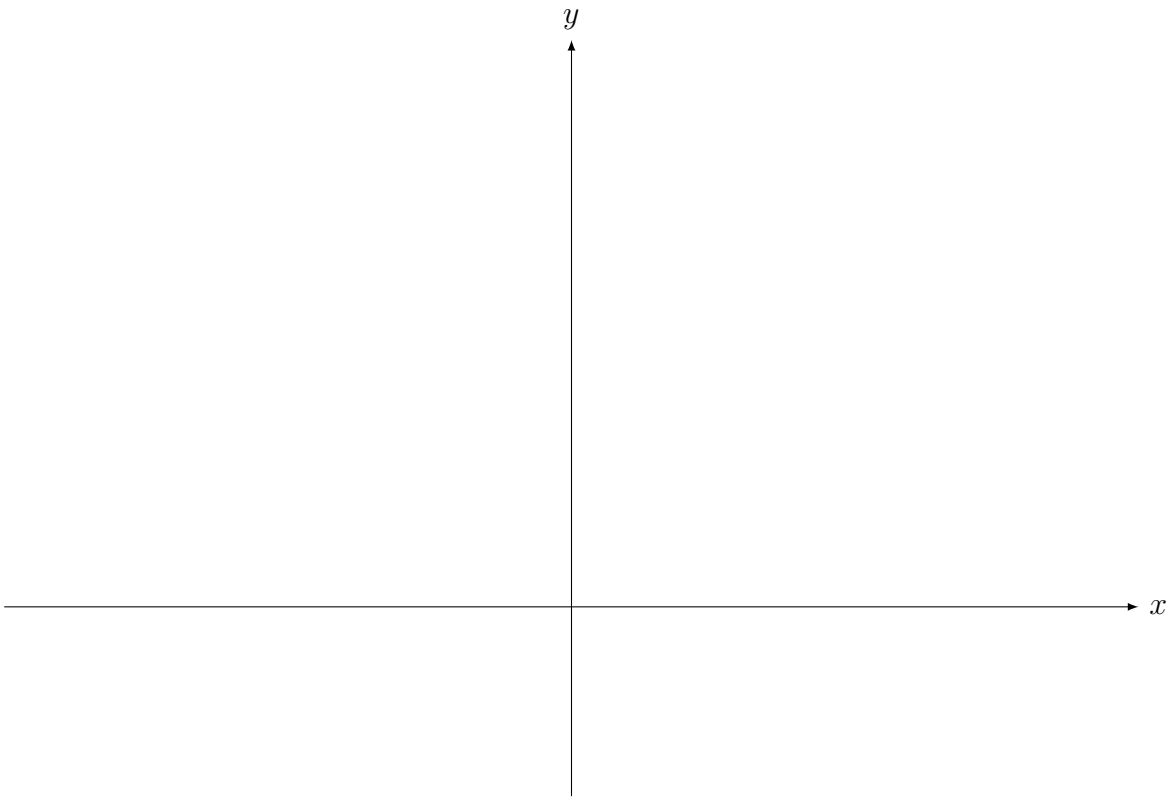
   (b) $\tan(\alpha) =$

   (c) $\sec(\alpha) =$

   (d) $\csc(\alpha) =$

   (e) $\cot(\alpha) =$

4. Graph $y = 3\cos(\frac{1}{2}\pi x - \pi) + 3$.



5. Find the exact value of each in radians, if any value is undefined write "undefined":

   (a) $\arcsin(-1) =$

   (b) $\sec^{-1}(\sqrt{3}) =$

   (c) $\tan^{-1}(-1) =$

   (d) $\cos^{-1}(\cos(\frac{7\pi}{4})) =$

(e) $\sin(\sin^{-1}(\frac{\sqrt{3}}{2})) =$

(f) $\tan(\arcsin(-\frac{1}{2})) =$

(g) $\csc(\tan^{-1}(0)) =$

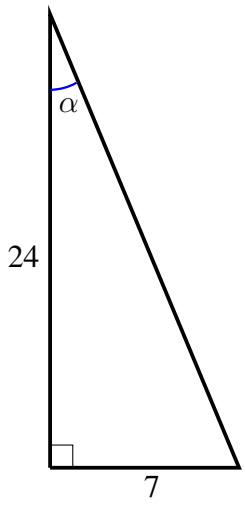
6. Find the inverse of the function and state its domain.

$$f(x) = \frac{1}{2}\cos(3x) - 1, \quad \text{for } 0 \leq x \leq \frac{\pi}{3}$$

(a) $f^{-1}(x) =$

(b) Domain of $f^{-1}(x)$:

7. For the given triangle find the inticated trigononetric function values



(a) $\sin(\alpha) =$

(b) $\cos(\alpha) =$

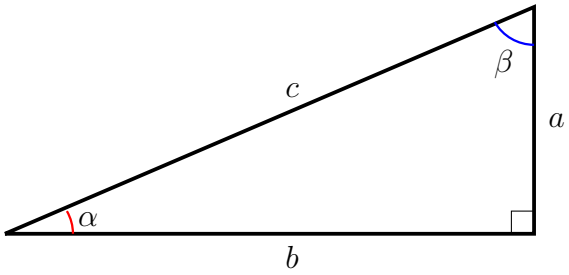
(c) $\tan(\alpha) =$

(d) $\sec(\alpha) =$

(e) $\csc(\alpha) =$

(f) $\cot(\alpha) =$

8. Solve the right triangle shown, where $a = 2$ and $b = 7$.



(a) $c =$

(b) $\alpha =$

(c) $\beta =$

7 Trigonometric Identities

1. For each of the following express as sines and cosines then use any identities to simplify.

   (a) $\sin^4 x - \cos^4 x$

   (b) $(1 + \sin x)(1 - \csc x)$

2. For the following, use identities to find the exact values for the remaining five trigonometric functions.

$$\tan\alpha = -\frac{8}{15}, \quad \frac{\pi}{2} < \alpha < \pi.$$

(a) $\sin\alpha =$

(b) $\cos\alpha =$

(c) $\tan\alpha =$

(d) $\sec\alpha =$

(e) $\csc\alpha =$

3. Determine if $f(x) = x - \sin x$ is symmetric to the y-axis, the origin, or $f(x)$ has no symmetry.

4. Verify the following identities:

   (a) $\ln|\csc x - \cot x| = -\ln|\csc x + \cot x|$

   (b) $\dfrac{1 - \tan^2 w + \sin^2 w \tan^2 w}{\sec^2 w} = \cos^4 w$

5. For each of the following equations find the solution set using the indicated units.

   (a) $\cos x = -0.9135$ (in degrees)

   (b) $\tan(2x) = \sqrt{3}$ (in radians)

6. For each equation find all solutions in the interval $[0, 2\pi)$ or $[0°, 360°)$ depending on the indicated units.

   (a) $4 \cdot 16^{\cos^2(x)} = 64^{\cos(x)}$ (in radians)

   (b) $9 \sec^2\theta \tan\theta = 12 \tan\theta$ (in radians)

(c) $\csc^4\theta - 5\csc^2\theta + 4 = 0$ (in degrees)