Instructions: This exam is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Remove any smartwatches and non-religious head-wear. Cheating of any kind will not be tolerated and will result in a grade of zero.**You must clear the memory on your calculator before beginning the exam.** Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. **Write your answers on the test with all work on a seperate piece of paper**. You have **2 hour 30 minutes** to finish the exam. Answer all questions to the best of your ability. Unless otherwise specified, you are required to **SHOW ALL YOUR WORK** to receive full credit. The exam has 220 possible points. You will be graded out of 200 points.

Questions	Possible	Score		Possible	Score
Question 1	16		Question 7	20	
Question 2	20		Question 8	20	
Question 3	20		Question 9	16	
Question 4	20		Question 10	20	
Question 5	12		Question 11	16	
Question 6	20		Question 12	16	
Extra Credit	4			Total	

1. (16 points) For this problem you need only give your solution unless otherwise stated by the question. All questions are worth 1 point except for (c) which is worth 2 points.

   (a) If $f''(a) < 0$, $f(x)$ is ____________ at a.

   (b) (T/F) If the demand curve and supply curve are linear, then consumer surplus equals producer surplus. ____________

   (c) Let $f'(3) = 0$, $f''(1) = 0$, and $f'(2) = 0$. Also, let $f'(x) < 0$ for $x < 2$ and $x > 3$, $f'(x) > 0$ for $2 < x < 3$, $f''(x) < 0$ for $x < 1$, and $f''(x) > 0$ for $x > 1$. Determine the local maxima(s), local minima(s), and inflection point(s). **Clearly label which is which**.

   (d) If $f(x)$ and $g(x)$ are continuous on $[a, c]$, $\int_a^b f(x)\,dx = 13$, and $\int_b^c f(x)\,dx = 37$, determine $\int_a^c f(x)\,dx$.

   (e) (T/F) If an exponential function $f(t)$ in terms of t has a constant percent increase r and intitial value 300, then $f(t) = 300r^t$. ______

   (f) If $A(n)$ is the area of a plot of horse farm land, in acres, necessary for n, the number of horses, interpret $A'(50) = .5$.

   (g) If an apple's diameter, d cm, is inversely proportional to the square of the number of apples in the tree ,a, then what is a formula, given constant of proportionality k, for the relationship?

   (h) (T/F) Let $(12, 50)$ be the break-even point for a given cost and revenue function with quantity 12 and dollar amount \$50. Then at a quantity of 13, the company is not making a profit. ______

   (i) Expand the following logarithm so there are no powers: $\ln\left(\frac{3x^4}{7\sqrt{y}}\right)$.

   (j) (T/F) $\frac{d}{dx}\left[h(k(x))\right] = h'(k'(x))$ ____________

   (k) (T/F) If marginal revenue is greater than marginal cost, the producer should produce and sell an additional unit. ____________

   (l) $\frac{d}{dx}a^x =$ ____________ where a is constant and not 0.

   (m) (T/F) $\int k \cdot f(x)\,dx = \int k\,dx \cdot \int f(x)\,dx$ where k is a constant. ____________

   (n) (T/F) $\int a(t)\,dt = \frac{d}{dt}(d(t))$ where $a(t)$ is the acceleration function, $v(0) = 0$, and $d(t)$ is the distance function. ____________

   (o) (T/F) If $f''(3) = 0$, then $x = 3$ is an inflection point. ____________

2. (5 points each) Find the first derivative of the following functions (simplify your answer completely!):

   (a) $f(x) = 3e^x + 6x^3 - 7\ln(x) - 6^x + e^2$

   (b) $f(x) = \ln(\pi^x)$

   (c) $f(x) = \frac{3}{2\sqrt[5]{x^2}}$

   (d) $\frac{e^x}{x^2}$

3. (5 points each) Evaluate the integrals.

   (a) $\displaystyle\int x(x+2)^6\,dx$

   (b) $\displaystyle\int_1^e \frac{\sqrt{\ln(x)}}{x}\,dx$

   (c) $\displaystyle\int -\frac{3}{x} + \frac{5}{2x^{1/3}} - e^x + 2\,dx$

   (d) $\displaystyle\int_0^{\ln(2)} \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

4. (20 points) Given the graph below where $A = (a, f(a))$, $B = (b, f(b))$, $C = (c, f(c))$, $D = (d, f(d))$, $E = (e, f(e))$, $G = (g, f(g))$, and $H = (h, f(h))$, find the following:

(a) Determine the local minima and local maxima of the graph. Label each.

(b) Find the intervals for which the function is increasing and the intervals for which the function is decreasing and label which are increasing and which are decreasing.

(c) Find the intervals for which the function is concave up and the intervals for which the function is concave down and label which are concave up and which are concave down.

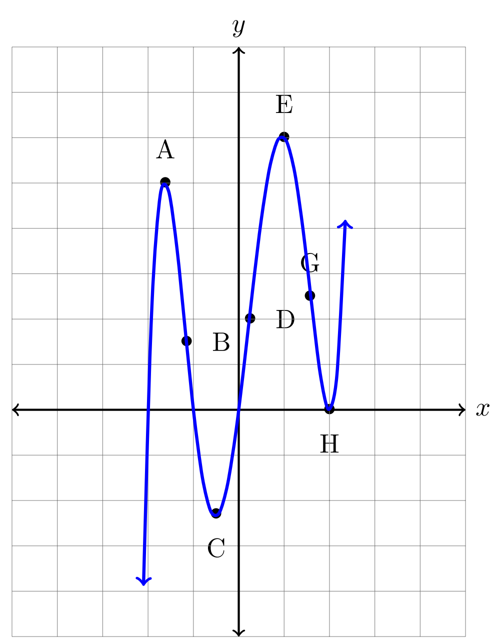
(d) Find all inflection points on the graph.

(e) On the interval $[a, d]$, determine the global maximum and minimum.

(f) On the interval $[b, d]$, determine the global maximum and minimum.

(g) On the interval $[d, g]$, determine the global maximum and minimum.

(h) On the interval $[a, h]$, determine the global maximum and minimum.



5. (12 points) Given the graph below of $f(x)$ find the following:

(a) $\int_0^2 f(x)\,dx$

(b) $\int_2^3 f(x)\,dx$

(c) $\int_0^4 f(x)\,dx$

(d) $\int_4^5 f(x)\,dx$

(e) $\int_5^6 f(x)\,dx$

(f) $\int_0^6 |f(x)|\,dx$

(g) $\int_6^8 f(x)\,dx$

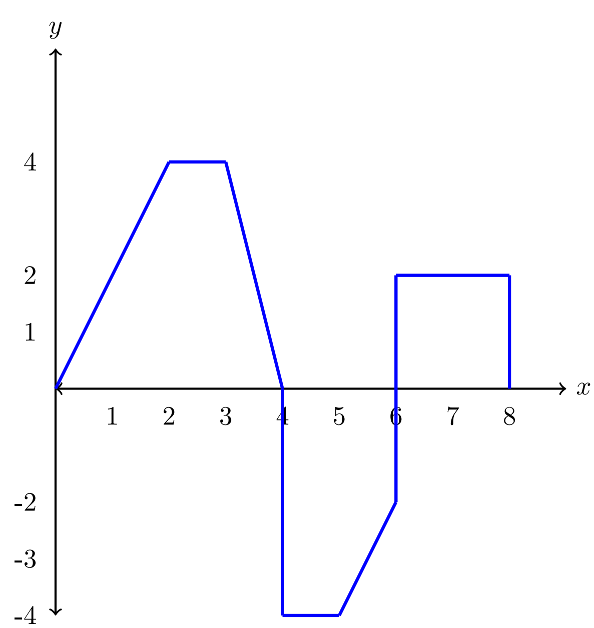
(h) $\int_4^6 f(x)\,dx$

(i) $\int_4^8 f(x)\,dx$

(j) $\int_4^8 |f(x)|\,dx$

(k) $\int_0^8 |f(x)|\,dx$

(l) $\int_0^8 f(x)\,dx$



6. (20 points) Jack sold his family's cow for a small sack of "magic beans" from a travelling merchant. He plants the beans and a large beanstalk begins to grow. He measures each day how tall the beans stalk is growing. Jack was astonished that after 2 days the beanstalk was 50 feet tall! Jack continued to be amazed since, on day 4, the beanstalk was 200 feet tall!

   (a) At the end of day 4, Jack is curious at what rate the beanstalk grew since day 2. Find the average rate of change so Jack can see the growth rate occurring.

   (b) Interpret the above result for Jack.

   (c) Let the two points lie on a tangent line to the actual height function of the beanstalk, $h(d)$, at $d = 3$ days. Find a tangent line and approximate for Jack the height of the beanstalk after 6 days.

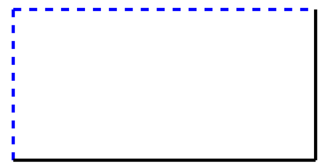
   (d) Jack continues to measure the height his beanstalk daily and puts together the following table:

d	0	2	4	6	8
$h(d)$	12.5	50	200	800	3200

   Determine if $h(d)$ is a linear function. If it is, find the function. If it is not, determine what kind of function it is and find the function.

7. (20 points) Solve the following word problems.

   (a) Mary wants to build a rectangular fenced area next to the interestion of two rivers so her little lambs do not need her to bring water. The area she found looks like the below with dashed lines representing the riversides.

   If Mary has 1000 feet of fencing, what is the maximum area of the fenced area?

   (b) Mary realized the above area was a safety hazard for her lambs since they could fall in the river. Now she wants to construct a fenced rectangle of area 312500 square feet in the rainforest of Costa Rica, where the air is so humid it can act as their water, and away from any hazardous rivers (She does not realize the abundant hazards of the rainforest; dumb Mary). She does realize however that it will cost more for her to get fencing and is now concerned about this cost. For the first 3 sides, it costs \$8 per foot and there is a discount on the last side of \$2 per foot. What is minimum cost of the fencing?

8. (20 points) Bart Simpson is riding his skateboard from home to the Noiseland Arcade with an acceleration given by $a(t) = 12t^2 - 6t + 2$ in feet per square minute where t is in minutes. If it takes Bart 10 minutes to get to the arcade and Bart is not moving prior to his ride, find the following:

   (a) Estimation of $\int_0^{10} v(t)\,dt$ using Left-hand Sums with $n = 2$ subdivisions (**include units**).

   (b) Estimation of $\int_0^{10} v(t)\,dt$ using Right-hand Sums with $n = 2$ subdivisions (**include units**).

   (c) Find a more accurate estimation using the two answers above (**include units**).

   (d) Find a formula the distance when Bart has not left his house at time 0.

   (e) Find the exact distance Bart travelled by integration and by evaluating the function found above at $t = 10$ (**include units**).

   (f) Interpret the meaning of the above answer.

9. (16 points) Peter Parker and his Uncle Ben own a clothing store specializing in silk shirts called A Spider's Choice. The total cost for q shirts, in dollars, is given by $C(q) = 460q^2 + 350q + 1000$ and the total revenue for q shirts, in dollars, is given by $R(q) = 500q^2 - 50q + 2000$.

(a) Find the profit when producing 15 shirts.

(b) Find the break-even point for the the business.

(c) Should the two produce the 16th shirt? Explain.

(d) Estimate the change in profit when producing the 16th shirt.

(e) At what quantity is revenue maximized?

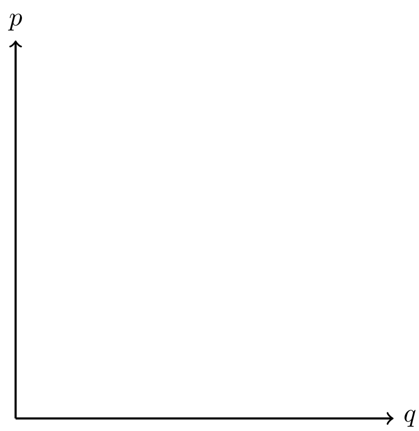
(f) At what quantity is profit maximized?

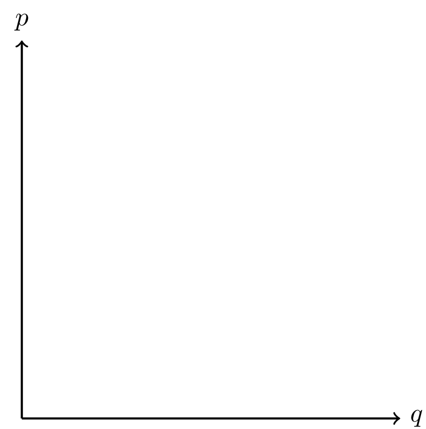
10. (20 points) Draw a graph of each of the market models and label the following:

- equilibrium $(q*, p*)$.
- p-intercepts
- Consumer Surplus with value (use C.S.)
- Producer Surplus with value (use P.S.)

(a) Let the market be given by $D(q) = -3q + \frac{39}{2}$ and $S(q) = 3q + \frac{3}{2}$.

(b) Let the market be given by $D : \frac{1}{2}p + 4q = 15$ and $S : \frac{1}{3}p - q = \frac{8}{3}$.





11. (16 points) Let Duncan Black invest \$3000 in an account compounded monthly at a rate of 3% and let Alonzo Decker invest \$3000 in an account compounded continuously at a rate of 2.5%.

   (a) Find an function for the amount in the account, P, in terms of years, t, for each account.

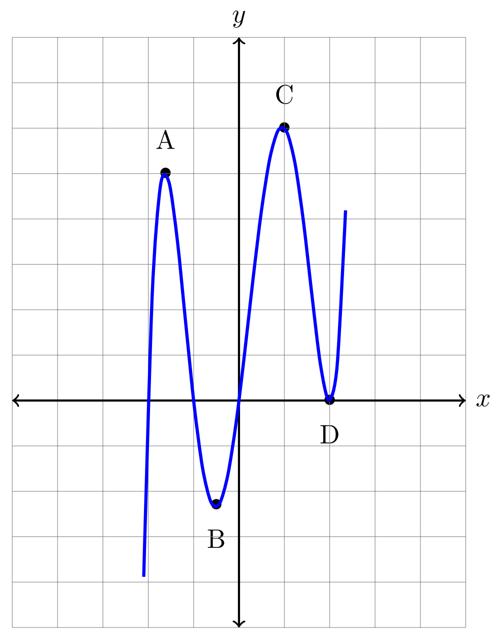
   (b) Who has more in their account at the end of 5 years? (**Show how you got your answer**)

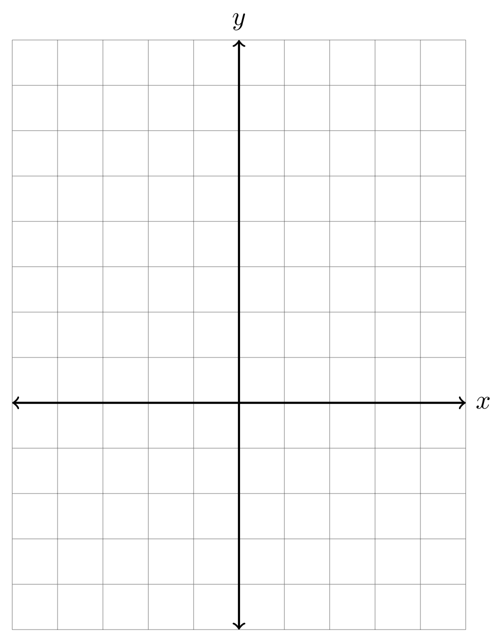
   (c) Find the time at which the value of each account doubles. (**Show how you got your answer**)

   (d) If the bank gave Alonzo a better rate of 3%, who has more in their account after 5 years? (There are two ways to answer this question. Both require an explanation/work)

12. (16 points)

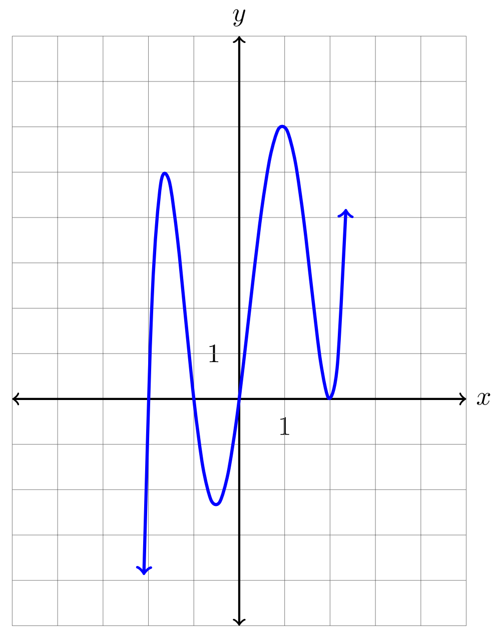
(a) Given the following graph for $f(x)$, sketch the derivative, $f'(x)$.





(b) Given the following **graph for** $f'(x)$, determine the following for $f(x)$:

- All the critical points
- Local maxima and local minima
- Increasing and decreasing intervals



Extra Credit (4 points): If you could choose a superpower what would it be? Now draw yourself using your superpower.