

Name: \_\_\_\_\_

Exam 3 (Version A)

**Instructions:** This exam is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Remove any smartwatches and non-religious head-wear. Cheating of any kind will not be tolerated and will result in a grade of zero. **You must clear the memory on your calculator before beginning the exam.** Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. **Write your answers on the test with all work on a separate piece of paper.** You have **1 hour 40 minutes** to finish the exam. Answer all questions to the best of your ability. Unless otherwise specified, you are required to **SHOW ALL YOUR WORK** to receive full credit. The exam has 110 possible points. You will be graded out of 100 points.

Questions	Possible	Score		Possible	Score
Question 1	12		Question 5	10	
Question 2	18		Question 6	16	
Question 3	16		Question 7	14	
Question 4	10		Question 8	12	
Extra Credit	2			Total	

1. (12 points) For this problem you need only give your solution unless otherwise stated by the question. Part (g) is worth 2 points. All others are worth 1 point.

(a) If  $F(x)$  is the antiderivative of  $f(x)$  continuous on  $[a, b]$  then  $\int_a^b f(x) dx = \underline{\hspace{2cm}}$ .

(b) If  $h(n)$  is the rate of height of a building in New York in feet per window and  $n$  is the number of windows, interpret  $\int_0^{100} h(n) dx = 600$ .

(c) (T/F) The Right-hand sum is always an overestimate and the Left-hand sum is always an underestimate.  $\underline{\hspace{2cm}}$

(d) (T/F) For  $k$  constant,  $\int k dx = 0$ .  $\underline{\hspace{2cm}}$

(e) If  $f(x)$  and  $g(x)$  are continuous on  $[a, b]$ ,  $\int_a^b f(x) dx = 2$ , and  $\int_b^a g(x) dx = 7$ , determine  $\int_a^b (f(x) + g(x)) dx$ .

(f) (T/F) Consumer surplus is the amount gained by a consumer by buying at retail rather than the maximum they are willing to pay.  $\underline{\hspace{2cm}}$

(g) Explain the difference between an indefinite integral and a definite integral. Then give an example of each.

(h) State the Power Rule for an indefinite integral.  $\underline{\hspace{2cm}}$

(i)  $\int \frac{1}{x} dx = \underline{\hspace{2cm}}$

(j) (T/F)  $\int u'(x) \cdot f(u(x)) dx = \int f(u) du$   $\underline{\hspace{2cm}}$

(k) (T/F)  $\int f(x) dx = \frac{LHS + RHS}{2}$   $\underline{\hspace{2cm}}$

2. (3 points each) Evaluate the integrals:

(a)  $\int e^x + \frac{2}{x} - \frac{12}{2\sqrt[3]{x}} dx$

(b)  $\int_0^3 3x^4 - 7x^3 - 9x^2 + 10x - 1 dx$

(c)  $\int_{\frac{-4}{11}}^{\frac{4}{11}} dx$

(d)  $\int_0^{\sqrt{2}} \frac{8x}{(1-x^2)^4} dx$

(e)  $\int_7^8 x(x-7)^8 dx$

(f)  $\int e^{t+e^t} dt$

3. (16 points) Ash is walking from Pallet Town to Viridian City with an acceleration given by  $a(t) = 3t^2 - 5t + 2$  in yard per minute squared where  $t$  is in minutes. If it takes Ash 20 minutes to get to Viridian City and Ash is not moving prior to his trip, find the following:

(a) Estimation of  $\int_0^{20} v(t) dt$  using Left-hand Sums with  $n = 5$  subdivisions (**include units**).

(b) Estimation of  $\int_0^{20} v(t) dt$  using Right-hand Sums with  $n = 5$  subdivisions (**include units**).

(c) Find a more accurate estimation using the two answers above (**include units**). (Hint: What is the most accurate estimation we know besides using antiderivatives?)

(d) Find a formula the distance when Ash is still in Pallet Town at time 0.

(e) Find the exact distance Ash travelled by integration and by evaluating the function found above at  $t = 20$  (**include units**).

(f) Interpret the meaning of the above answer.

4. (10 points) Given the graph below of  $f(x)$  find the following:

(a)  $\int_0^1 f(x) dx$

(b)  $\int_0^2 f(x) dx$

(c)  $\int_2^4 f(x) dx$

(d)  $\int_4^5 f(x) dx$

(e)  $\int_5^7 f(x) dx$

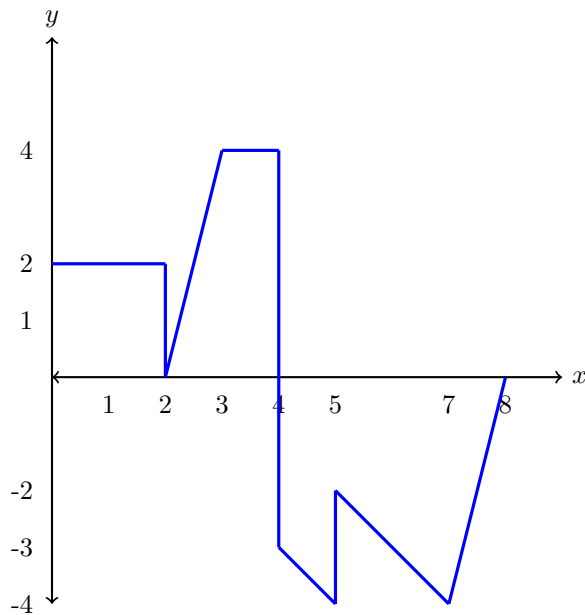
(f)  $\int_7^8 f(x) dx$

(g)  $\int_0^4 f(x) dx$

(h)  $\int_4^8 f(x) dx$

(i)  $\int_0^8 f(x) dx$

(j)  $\int_0^8 |f(x)| dx$



5. (10 points) Use the graph of  $f(x)$  below to estimate the following not using left, right or average sums: (Hint: Use the grids of the graph)

(a)  $\int_{-3}^{-1} f(x) dx$

(b)  $\int_{-1}^{-2} f(x) dx$

(c)  $\int_{-1}^0 f(x) dx$

(d)  $\int_0^1 f(x) dx$

(e)  $\int_1^3 f(x) dx$

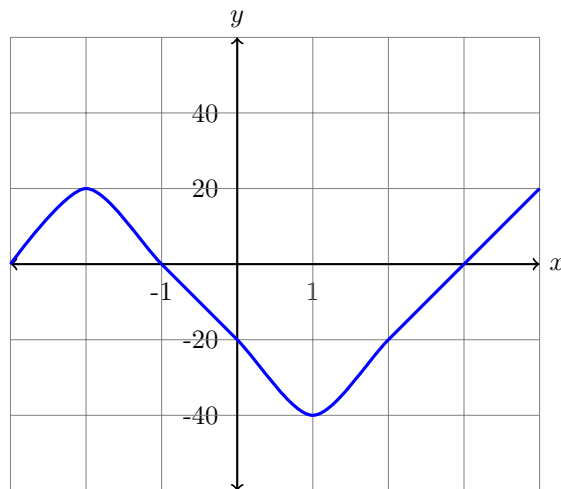
(f)  $\int_{-1}^3 f(x) dx$

(g)  $\int_3^4 f(x) dx$

(h)  $\int_{-3}^3 |f(x)| dx$

(i)  $\int_{-3}^4 f(x) dx$

(j)  $\int_{-3}^4 |f(x)| dx$

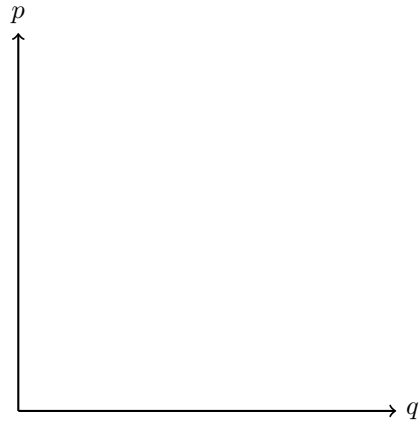
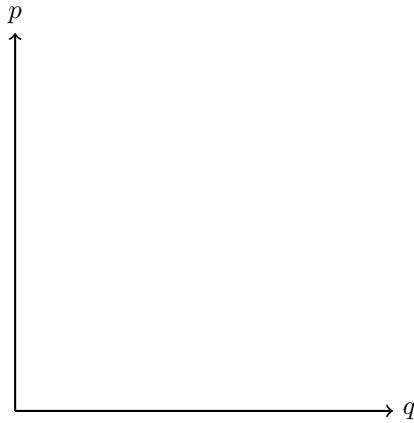


6. (16 points) Draw a graph of each of the market models and label the following:

- equilibrium  $(q^*, p^*)$ .
- $p$ -intercepts
- Consumer Surplus with value (use C.S.)
- Producer Surplus with value (use P.S.)

(a) Let the market be given by  $D(q) = -2q + \frac{41}{2}$  and  $S(q) = 3q + \frac{1}{2}$ .

(b) Let the market be given by  $D : \frac{1}{2}p + \frac{3}{2}q = 9$  and  $S : \frac{1}{2}p - q = 1$ .



7. (14 points) Mario and Luigi are competing to save Princess Peach. Whoever gets to Princess Peach first gets to stop being a plumber and becomes king (As we all know this never lasts long...). Let table 1 represent the velocity of Mario  $v_m(t)$  in meters per minute with  $t$  in minutes and table 2 represent the velocity of Luigi  $v_l(t)$  in meters per minute with  $t$  in minutes.

(a) Table 1.

$t$	20	23	26	29	32	35	38
$v_m(t)$	62	72	87	92	7	666	333

(b) Table 2.

$t$	15	$15\frac{1}{2}$	16	$16\frac{1}{2}$	17	$17\frac{1}{2}$	18
$v_l(t)$	554	881	1280	1177	2019	1234	1

We may assume they both arrived at the same time since they started their adventures at different times (AKA We may assume they weren't moving in the time before the first blocks of the table). Since they arrived at the same time, the winner of the throne will be decided by who took the shortest path to Princess Peach. Answer the following questions to determine the winner of the throne.

- (a) (3 points each) Find left-hand sums for the distance travelled by Mario and Luigi (Label each answer as Mario or Luigi with units).
- (b) (3 points each) Find right-hand sums for the distance travelled by Mario and Luigi (Label each answer as Mario or Luigi with units).
- (c) (1 point each) Using the above, come up with the best estimation we know for each and use them to determine who gets to take the throne and who goes back to being a plumber. (Label each answer as Mario or Luigi with units).



8. (12 points)

(a) Find the area, when  $x \geq 0$ , between the two curves:

$$f(x) = x^3 + 1 \qquad g(x) = x^2 + 2x + 1$$

(b) Find the area, when  $x \leq 0$ , between the two curves:

$$f(x) = x^3 + 1 \qquad g(x) = x^2 + 2x + 1$$

9. Extra Credit (2 points): Draw your spirit animal with an explanation as to why it is your spirit animal.