Name: **Exam 2 (Version A)**

Instructions: This exam is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Remove any smartwatches and non-religious head-wear. Cheating of any kind will not be tolerated and will result in a grade of zero.**You must clear the memory on your calculator before beginning the exam.** Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. Write your answers on the test. You have **1 hour 40 minutes** to finish the exam. Answer all questions to the best of your ability. Unless otherwise specified, you are required to **SHOW ALL YOUR WORK** to receive full credit. The exam has 108 possible points. You will be graded out of 100 points.

Questions	Possible	Score		Possible	Score
Question 1	12		Question 5	12	
Question 2	12		Question 6	16	
Question 3	12		Question 7	16	
Question 4	12		Question 8	12	
Extra Credit	4			Total	

1. (12 points) For this problem you need only give your solution unless otherwise stated by the question. Part (g) is worth 2 points. All others are worth 1 point.

   (a) If $f'(a) > 0$ then $f(x)$ is ____________ at a.

   (b) If $H(n)$ is the height of a building in New York in feet and n is the number of windows, interpret $H'(100) = 5$.

   (c) (T/F) Acceleration is the derivative of velocity. ____________

   (d) (T/F) The second derivative at a, $f''(a)$, determines if $f(x)$ is increasing or decreasing at a. ____________

   (e) If $C'(100) = 5.2$ and $R'(100) = 6$, determine the change in profit for producing and selling the 101^{st} unit.

   (f) (T/F) Marginal revenue at q quantity is the slope of the secant line at q of the renvenue function.____________

   (g) Let $f'(1) = 0$, $f''(0) = 0$, and $f'(\ -1) = 0$. Also, let $f'(x) < 0$ for $x < -1$ and $x > 1$, $f'(x) > 0$ for $-1 < x < 1$, $f''(x) < 0$ for $x < 0$, and $f''(x) > 0$ for $x > 0$. Determine the local maxima(s), local minima(s), and inflection point(s). **Clearly label which is which**.

   (h) State the Power Rule for $f(x) = x^n$. ____________

   (i) $\frac{d}{dx}\Big(\ln(x)\Big) =$ ____________

   (j) (T/F) $\frac{d}{dx}\left[\frac{h(x)}{k(x)}\right] = \frac{h'(x)k(x) - k'(x)h(x)}{k^2(x)}$ ____________

   (k) (T/F) $\frac{d}{dx}\Big[h(k(x))\Big] = h'(k'(x))$ ____________

2. (2 points each) Find the first and second derivative for the following functions:

(a) $f(x) = x^3 - \frac{3}{2}x^2 + 13^2$

(b) $f(x) = 3\ln(x) + 2x^2 - 8\mathrm{e}^x + 2^x$

(c) $f(x) = \mathrm{e}^{2x^2}$

(d) $f(x) = 2x\ln(x^2)$

(e) $f(x) = \dfrac{12}{5\sqrt[3]{x}}$

(f) $f(x) = \dfrac{e^x}{x}$

3. (12 points) John, Jacob, and Jingleheimer are all brothers who own The Schmidt Brothers Sandwich Shop. The total cost for q sandwiches, in dollars, is given by $C(q) = -3q^2 + 4q + 10$ and the total revenue for q sandwiches, in dollars, is given by $R(q) = -4q^2 = 26q$.

(a) Find the profit when producing 20 sandwiches.

(b) Should the brothers produce the 21st sandwich? Explain.

(c) Estimate the change in profit when producing the 21st sandwich.

(d) At what quantity is revenue maximized?

(e) At what quantity is profit maximized?

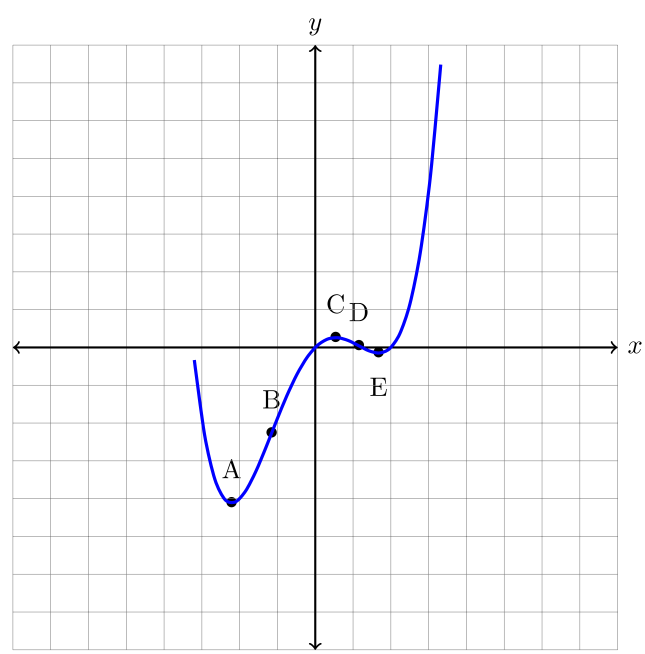
4. (12 points) Given the graph below where $A = (a, f(a))$, $B = (b, f(b))$, $C = (c, f(c))$, $D = (d, f(d))$, and $E = (e, f(e))$, find the following:

(a) Determine the local minima and local maxima of the graph. Label each.

(b) Find the intervals for which the function is increasing and the intervals for which the function is decreasing and label which are increasing and which are decreasing (Note: Use open brackets for these intervals).

(c) Find the intervals for which the function is concave up and the intervals for which the function is concave down and label which are concave up and which are concave down (Note: Use open brackets for these intervals).

(d) Find all inflection points on the graph.



5. (12 points) Let $f(x)$ be a function. Let (2,5) and (-2,1) be points on the tangent line of $f(x)$ at $x = 2$.

   (a) Find $f'(2)$ and give an equation for the tangent line to $f(x)$ at $x = 2$.

   (b) Using the above, approximate $f(5)$. (This is a tangent line approximation)

   (c) Explain what a tangent line is in your own words.

6. (16 points) Solve the following world problems.

   (a) Me and Julio want to build a rectangular fenced area down by the schoolyard. The rectangular area will share a fence with the schoolyard and thus one side is already fenced. If we have 5000 feet of fencing, what is the maximum area of the fenced area?

   (b) Julio and I have changed our minds. Now we want to construct the fenced area in a rectangle of area 5000 square feet in the middle of nowhere We are not the best farmers... We are now more concerned about the cost of fencing. For the first 3 sides, it costs $50 per foot and there is a discount on the last side of $25 per foot. What is minimum cost of the fencing?

Extra Credit (4 points): Draw a picture of both of the fenced areas and a reason why we moved locations! (Be super creative!)

7. (16 points)

(a) Let the function, $f(x)$, have the folllowing derivative:

$$f'(x) = 2x^2(x+3)^3(x-1)(x-5)$$

Fill in the following table and determine the minimum and maximum values. **SHOW ALL WORK.**

Table 1.

	$(x+3)^3$	$2x^2$	$(x-1)$	$(x-5)$	$f'(x)$
$x < -3$					
$-3 < x < 0$					
$0 < x < 1$					
$1 < x < 5$					
$x > 5$					

(b) Let the function, $f(x)$, have the folllowing derivative:

$$f''(x) = x(x+1/2)^2(x-1)^2(x-4)$$

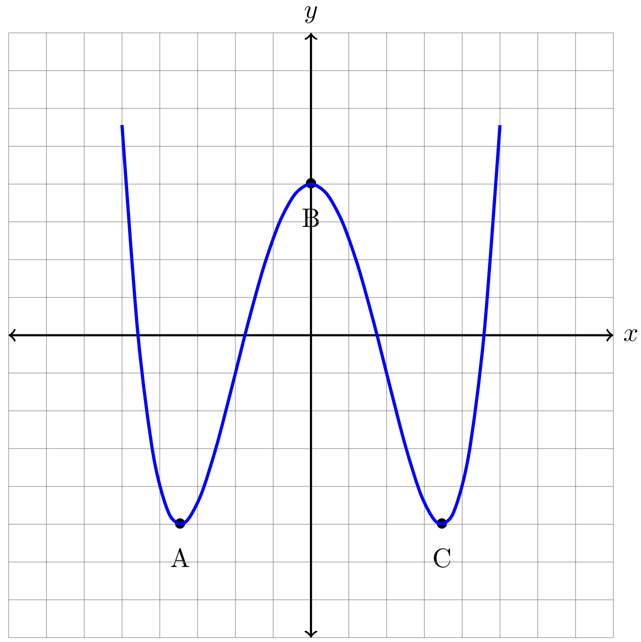
Fill in the following table and determine the intervals of concavity. **SHOW ALL WORK.**

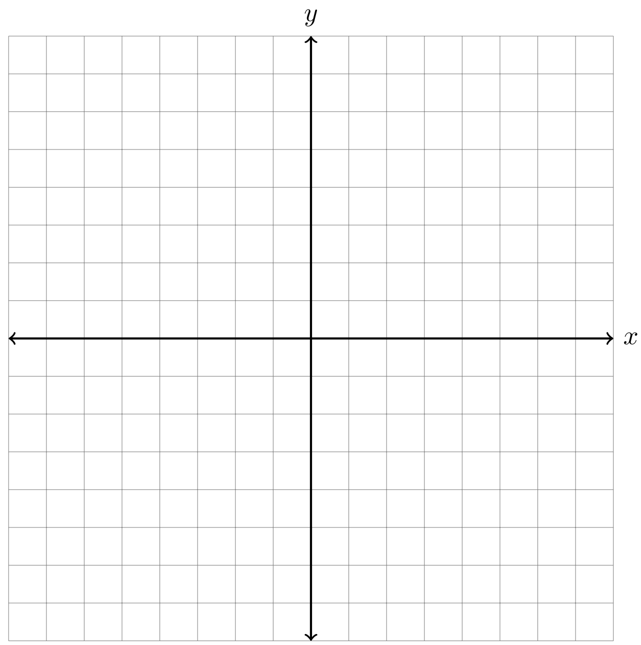
Table 2.

	$(x+1/2)^2$	x	$(x-1)^2$	$(x-4)$	$f''(x)$
$x < -1/2$					
$-1/2 < x < 0$					
$0 < x < 1$					
$1 < x < 4$					
$x > 4$					

8. (12 points)

(a) Given the following graph for $f(x)$, sketch the derivative, $f'(x)$.





(b) Given the following graph for $f'(x)$, give information about $f(x)$ (This includes increasing/decreasing, maxima minimum, etc.).