Instructions: This homework is an individual effort. Answer each question. This is due on **Monday, April 20th**. **Show all work to receive full credit.**

1. Evaluate the following integrals:

a. $\int x(1-5x^2)^5\,dx$

Solution 1. $u = 1 - 5x^2$

$$du = -10x dx \Rightarrow \frac{du}{-10} = x dx$$

$$\int x(1-5x^2)^5\,dx = \int -\frac{u^5}{10}\,du = \frac{-u^6}{60} + C = -\frac{(1-5x^2)^6}{60} + C$$

b. $\int \frac{\sqrt{\ln(x)}}{x}\,dx$

Solution 2. $u = \ln(x)$

$$du = \frac{dx}{x}$$

$$\int \frac{\sqrt{\ln(x)}}{x}\,dx = \int \sqrt{u}\,du = \frac{2u\sqrt{u}}{3} + C = \frac{2\ln(x)\sqrt{\ln(x)}}{3} + C$$

c. $\int 6qe^{q^2+1}\,dq$

Solution 3. $u = q^2 + 1$

$$du = 2qdq \Rightarrow 3du = 6qdq$$

$$\int 6qe^{q^2+1}\,dq = \int 3e^u\,du = 3e^u + C = 3e^{q^2+1} + C$$

d. $\int \frac{4x^3}{x^4+1}\,dx$

Solution 4. $u = x^4 + 1$

$$du = 4x^3 dx$$

$$\int \frac{4x^3}{x^4+1}\,dx = \int \frac{du}{u}\,du = \ln|u| + C = \ln|x^4+1| + C$$

e. $\int \frac{e^t}{e^t+5}\,dt$

Solution 5. $u = e^t + 5$

$$du = e^t dt$$

$$\int \frac{e^t}{e^t+5}\,dt = \int \frac{du}{u}\,du = \ln|u| + C = \ln|e^t+5| + C$$

f. $\displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$

Solution 6. $u = e^x + e^{-x}$

$du = (e^x - e^{-x})dx$

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx = \int \frac{du}{u}\,du = \ln|u| + C = \ln|e^x + e^{-x}| + C$$

g. $\displaystyle\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy$

Solution 7. $u = \sqrt{y}$

$$du = \frac{1}{2\sqrt{y}}dy \Rightarrow 2du = \frac{1}{\sqrt{y}}dy$$

$$\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy = \int 2e^u\,du = 2e^u + C = 2e^{\sqrt{y}} + C$$

h. $\displaystyle\int \frac{x+1}{x^2+2x+19}\,dx$

Solution 8. $u = x^2 + 2x + 19$

$$du = (2x+2)dx \Rightarrow \frac{du}{2} = (x+1)dx$$

$$\int \frac{x+1}{x^2+2x+19}\,dx = \int \frac{1}{2u}\,du = \frac{\ln|u|}{2} + C = \frac{\ln|x^2+2x+19|}{2} + C$$

i. $\displaystyle\int_7^8 x(x-7)^8\,dx$

Solution 9. $u = x - 7 \Rightarrow x = u + 7$

$du = dx$

$$\int_7^8 x(x-7)^8\,dx = \int (u+7)u^8\,du = \int (u^9 + 7u^8)\,du = \frac{u^{10}}{10} + \frac{7u^9}{9}\Big|$$

$$= \frac{(x-7)^{10}}{10} + \frac{7(x-7)^9}{9}\Big|_7^8 = \frac{1}{10} + \frac{7}{9}$$

2. Suppose we have the inverse demand function $p = D(q) = 100 - 4q$, and suppose that equilibrium quantity $q^* = 5$. Determine the consumer surplus.

Solution 10. $p* = 100 - 4(5) = 80$

$$C.S. = \left(\frac{1}{2}\right)(5)(D(0) - 80) = \left(\frac{1}{2}\right)(5)(100 - 80) = 50$$

3. Suppose we have the inverse demand supply $p = D(q) = 35 - q$ and the inverse supply function $p = S(q) = 3 + q$. Determine the producer and consumer surplus.

Solution 11. $3 + q = 35 - q \Rightarrow 2q = 32 \Rightarrow q* = 16 \Rightarrow p* = 19$

$$C.S. = \left(\frac{1}{2}\right)(16)(35 - 19) = \left(\frac{1}{2}\right)(16)(16) = 128$$

$$P.S. = \left(\frac{1}{2}\right)(16)(19 - 3) = \left(\frac{1}{2}\right)(16)(16) = 128$$

4. Suppose we have the supply function $q = S(p) = 10p - 30$ and demand function $q = D(p) = 30 - 2p$. Determine the producer and consumer surplus.

Solution 12. $10p - 30 = 30 - 2p \Rightarrow 12p = 60 \Rightarrow p* = 5 \Rightarrow q* = 20$

$$C.S. = \left(\frac{1}{2}\right)(20)(15 - 5) = \left(\frac{1}{2}\right)(20)(10) = 100$$

$$P.S. = \left(\frac{1}{2}\right)(20)(5 - 3) = \left(\frac{1}{2}\right)(20)(2) = 20$$