Instructions: This homework is an individual effort. Answer each question. This is due on **Monday, April 6th**. **Show all work to receive full credit.**

1. Approximate the following integrals using (i) a left-hand sum with $n = 4$ subdivisions; and then (ii) a right-hand sum with $n = 4$ subdivisions; and then use these to get a better approximation. Does the left-hand sum overestimate or underestimate the integral? What about the right-hand sum?

a. $\displaystyle\int_1^3 e^x\,dx.$

Solution 1. $\Delta x = \frac{3-1}{4} = \frac{1}{2}$
$LHS = \frac{1}{2}(f(1) + f(1.5) + f(2) + f(2.5)) = \frac{1}{2}(e + e^{1.5} + e^2 + e^{2.5}) = 13.3857...$
$RHS = \frac{1}{2}(f(1.5) + f(2) + f(2.5) + f(3)) = \frac{1}{2}(e^{1.5} + e^2 + e^{2.5} + e^3) = 22.0693...$
LHS underestimate and RHS overestimate.

b. $\displaystyle\int_{-3}^{-1} x^2\,dx.$

Solution 2. $\Delta x = \frac{-1+3}{4} = \frac{1}{2}$
$LHS = \frac{1}{2}(f(-3) + f(-2.5) + f(-2) + f(-1.5)) = \frac{1}{2}((-3)^2 + (-2.5)^2 + (-2)^2 + (-1.5)^2) = 10.75$
$RHS = \frac{1}{2}(f(-2.5)+f(-2)+f(-1.5)+f(-1)) = \frac{1}{2}((-2.5)^2+(-2)^2+(-1.5)^2+(-1)^2) = 6.75$
RHS underestimate and LHS overestimate.

c. $\displaystyle\int_1^3 \ln(x)\,dx.$

Solution 3. $\Delta x = \frac{3-1}{4} = \frac{1}{2}$
$LHS = \frac{1}{2}(f(1) + f(1.5) + f(2) + f(2.5)) = \frac{1}{2}(\ln(1) + \ln(1.5) + \ln(2) + \ln(2.5)) = 1.007...$
$RHS = \frac{1}{2}(f(1.5) + f(2) + f(2.5) + f(3)) = \frac{1}{2}(\ln(1.5) + \ln(2) + \ln(2.5) + \ln(3)) = 1.556...$
LHS underestimate and RHS overestimate.

d. $\displaystyle\int_{-1}^{-3} \frac{1}{x}\,dx.$

Solution 4. *NOTE: Bound is flipped so negate all integral answers!* $\Delta x = \frac{-1+3}{4} = \frac{1}{2}$
$LHS = -\frac{1}{2}(f(-3) + f(-2.5) + f(-2) + f(-1.5)) = -\frac{1}{2}((1/-3) + (1/-2.5) + (1/-2) + (1/-1.5)) = .95$
$RHS = -\frac{1}{2}(f(-2.5) + f(-2) + f(-1.5) + f(-1)) = -\frac{1}{2}((1/-2.5) + (1/-2) + (1/-1.5) + (1/-1)) = 1.28$
LHS underestimate and RHS overestimate.

e. $\displaystyle\int_1^3 \sqrt{x}\,dx.$

Solution 5. $\Delta x = \frac{3-1}{4} = \frac{1}{2}$
$LHS = \frac{1}{2}(f(1) + f(1.5) + f(2) + f(2.5)) = \frac{1}{2}(\sqrt{1} + \sqrt{1.5} + \sqrt{2} + \sqrt{2.5}) = 2.610...$
$RHS = \frac{1}{2}(f(1.5) + f(2) + f(2.5) + f(3)) = \frac{1}{2}(\sqrt{1.5} + \sqrt{2} + \sqrt{2.5} + \sqrt{3}) = 2.976...$
LHS underestimate and RHS overestimate.

2. Consider the following table:

x	0	10	20	30	40
$f(x)$	350	410	435	450	460

a. Estimate $\int_0^{40} f(x)\,dx$ with a left-hand sum.

Solution 6. $LHS = 10(f(0)+f(10)+f(20)+f(30)) = 10(350+410+435+450) = 16450$

b. Estimate $\int_0^{40} f(x)\,dx$ with a right-hand sum.

Solution 7. $LHS = 10(f(10)+f(20)+f(30)+f(40)) = 10(410+435+450+460) = 17550$

c. Use the above to find a better approximation.

Solution 8. $B.A. = \frac{LHS+RHS}{2} = \frac{16450+17550}{2} = 17000$

3. After a foreign substance is introduced into the blood, the rate at which antibodies are made is given by $r(t) = \dfrac{t}{t^2+1}$ thousands of antibodies per minute, where t is in minutes. Interpret $\int_0^4 r(t)dt$.

Solution 9. $\int_0^4 r(t)dt$ is the amount of thousands of antibodies made over the first 4 minutes.

4. A forest fire is growing at a rate of $8\sqrt{t}$ acres per hour. Interpret $\int_0^{24} 8\sqrt{t}dt$.

Solution 10. $\int_0^{24} 8\sqrt{t}dt$ is the number of acres covered by fire after the first 24 hours.

5. Water is pumped out of a holding tank at a rate of $5-5e^{-0.12t}$ liters per minute, where t is the number of minutes since the pump started. Write an equation, $w(t)$, representing the amount of water pumped out after 1 hour.

Solution 11. $w(t) = \int_0^{60} (5-5e^{-.12t})dt$

6. Let $f(x)$ be given the below graph. Find the following:

a. $\int_0^3 f(x)dx$

Solution 12. $\int_0^3 f(x)dx = 3 \cdot 5 = 15$

b. $\int_3^6 f(x)dx$

Solution 13. $\int_3^6 f(x)dx = (1/2)(2+5)3 = \frac{21}{2}$

c. $\int_0^6 f(x)dx$

Solution 14. $\int_0^6 f(x)dx = \int_0^3 f(x)dx + \int_3^6 f(x)dx = 15 + \frac{21}{2}$

d. $\int_{10}^6 f(x)dx$

Solution 15. $\int_{10}^6 f(x)dx = -\int_6^{10} f(x)dx = -\left(\int_6^8 f(x)dx + \int_8^{10} f(x)dx\right)$

$= -((1/2)(2)(5) - (1/2)(2)(2)) = -(5-2) = -3$

e. $\int_0^{10} f(x)dx$

Solution 16. $\int_0^{10} f(x)dx = \int_0^6 f(x)dx - \int_{10}^6 f(x)dx = 15 + \frac{21}{2} - (-3) = 18 + \frac{21}{2}$

f. $\int_0^{10} |f(x)|dx$

Solution 17. $\int_0^{10} |f(x)|dx = \int_0^8 f(x)dx + \int_8^{10} |f(x)|dx = 15 + \frac{21}{2} + (1/2)(2)(5) + (1/2)(2)(2) = 22 + \frac{21}{2}$