Instructions: This homework is an individual effort. Answer each question. This is due on **Monday, March 23rd**. **Show all work to receive full credit.**

1. Find the quantity which maximizes profit when the total revenue and total cost are given by $R(q) = 5q - .003q^2$ and $C(q) = 300 + 1.1q$ for $0 \leq q \leq 1000$. Then find the quantity which minimizes profit.

   Solution 1. $\pi(q) = R(q) - C(q) = -.003q^2 + 3.9q - 300$
   $\pi'(q) = -.006q + 3.9 = 0$
   $q = 650$
   This is the maximum quantity since the derivative is positive to the right of 650 and negative to the left and the function is a down-facing parabola.

2. At a price of $80 for a half-day trip, a white-water rafting company attracts 300 customers. Every $5 decrease in price attracts an additional 30 customers.

   a. Find the demand equation $q = D(p)$ as a function of p price.

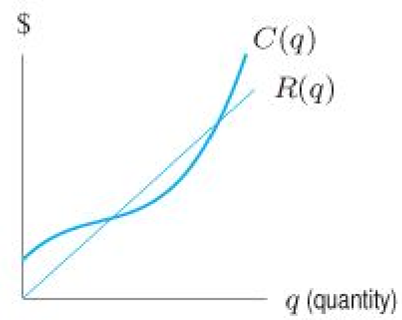
   b. Write an equation for revenue in terms of price p.

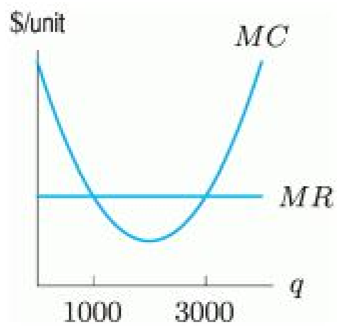
   c. What price should the company charge per trip to maximize revenue?

   Solution 2. $m = \frac{30}{-5} = -6$
   $q - q_1 = m(p - p_1) \rightarrow q - 300 = -6(p - 80)$
   $D(p) = q = -6p + 780$
   $R(q) = q \cdot p = D(p) \cdot p = -6p^2 + 780p = R(p)$
   $R'(p) = -12p + 780 = 0$
   $p = 65$
   This is the maximum price since the derivative is positive to the right of 65 and negative to the left and the function is a down-facing parabola.

3. a. Estimate where on the first graph maximum profit is by drawing a vertical line. Explain why you think this.

   b. Using the second graph of MC and MR, explain why your thought above is true in terms of derivatives!





Solution 3. (a) Where the cost is lowest in comparison to the revenue (middle of the graph) since profit is revenue minus cost.
(b) Since MC is the derivative of cost and similar for MR, we know marginal profit is marginal revenue minus marginal cost. Thus where the 2 intersect is the quantity for which marginal profit is 0. Thus, 3000 is the quantity needed to maximize profit. 1000 is not a maximum since marginal profit would be positive after 1000 not negative like a maximum would.

4. If $xy = 300$ for $x, y > 0$, find the minimum value of $x + y$.

Solution 4. $x = \frac{300}{y}$
$m = x + y = \frac{300}{y} + y$
$m' = \frac{-300}{y^2} + 1 = 0$
$y = \sqrt{300}$
$\rightarrow x = \sqrt{300}$ by the first line.
Thus, the minimum value of $x + y$ is $m = 2\sqrt{300}$.

5. Let Bob build a rectangular fence around 2000 ft^2 of land. The fence costs \$30 for the first 3 sides and \$10 for the last side. What is the minimum cost for Bob to build the enclosure? (Hint: Draw a picture! Come up with a cost equation and area equation!)

Solution 5. Let x be the length of two opposite sides of the rectangle and y be the length of the adjacent sides and C be the cost.

$$xy = 2000 \tag{1}$$

$$C = 30(2x + y) + 10y = 60x + 40y \tag{2}$$

Equation (1) gives us $x = \frac{2000}{y}$.
By substituting into equation (2), we get $C = \frac{120000}{y} + 40y$.
$C' = -\frac{120000}{y^2} + 40 = 0$
$y = \sqrt{3000}$
By equation (1), $x = \frac{2000}{\sqrt{3000}}$.
Thus, the minimum cost is $C = 60\left(\frac{2000}{\sqrt{3000}}\right) + 40(\sqrt{3000}) = \$4381.78...$

6. Let Bob build a farm next to a river. If Bob has 2000 ft of fencing, what is the maximum area of the enclosure? (Hint: Draw a picture! Use a perimeter equation.)

Solution 6. Let x be the length of two opposite sides of the rectangle and y be the length of the adjacent sides and A be the area.

$$2x + y = 2000 \tag{3}$$

$$A = xy \tag{4}$$

Equation (1) gives us $y = 2000 - 2x$.
By substituting into equation (2), we get $A = 2000x - 2x^2$.
$A' = 2000 - 4x = 0$
$x = 500$
By equation (1), $y = 1000$.
Thus, the maximum area is $A = 500 \cdot 1000 = 500000 ft^2$