Instructions: This portion of the exam is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Remove any smartwatches and non-religious head-wear. Cheating of any kind will not be tolerated and will result in a grade of zero.**You must clear the memory on your calculator before beginning the exam.** Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. You have **2 days** to finish this portion of the exam. Answer all questions to the best of your ability. Unless otherwise specified, you are required to **SHOW ALL YOUR WORK** to receive full credit. This portion of the exam has 165 possible points. You will be graded out of 156 points.

WRITE THIS PARAGRAPH ON WHAT YOU SUBMIT ALONG WITH A SIGNATURE AND DATE.

I, ———————————, will not under any circumstance use an online source, my notes, my peers, or any other resource besides my own knowledge and a calculator reset to factory settings to complete this exam. I will show all my work to demonstrate my knowledge on the topic.

Signature: ——————————— Date: ———————————

Questions	Possible	Score		Possible	Score
Question 1	20		Question 6	20	
Question 2	20		Question 7	20	
Question 3	20		Question 8	20	
Question 4	20				
Question 5	20		Extra Credit	5	
				Total	

1. (20 points) (If you get stuck at any step in this problem, email me and I will give you the answer, but you will recieve 0 points for that part of the question) Its your friend's birthday! Your best friend Albert decides to throw a suprise party. Of course Albert has to get all the essentials: cake, candles, decorations, and, most importantly, birthday hats! However, being the incredible mathematician he is, he notices that the base of the hats all have the same shape; parabolas! He does some quick comparing and decides that the upper half of the base is shaped like the curve, $y = -x^2 + 1$.
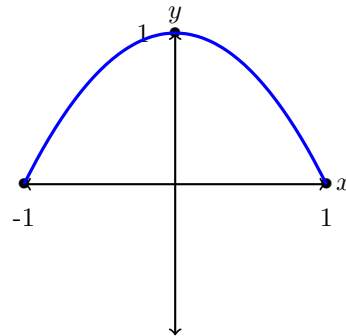
a. First, come up with the equation for the bottom half of the party hat based on the equation for the top half and draw it above.

b. Albert would like to find the area of the party hats! He is asking if you can find the area of the base of the hats (as given in your drawing). Find the area of the base of the each hat. (Hint: You have 2 curves. What is the area between them?)

c. While you were finding the area, Albert spent some time figuring out more information about the hat. He determined the volume of each hat is about proportional to a third of the area of the base with constant of proportionality: $k = h$ where h height of the hat. Using the above and the fact the hat is 6 inches tall, determine the area of each hat.

d. The volume of a circular cone is given as $V_c = \dfrac{\pi h r^2}{3}$. Is the hat an example of a circular cone? Explain.

2. (20 points) Jack sold his family's cow for a small sack of "magic beans" from a travelling merchant. He plants the beans and a large beanstalk begins to grow. He measures each day how tall the beans stalk is growing. Jack was astonished that after 2 days the beanstalk was 50 feet tall! Jack continued to be amazed since, on day 4, the beanstalk was 200 feet tall!

(a) At the end of day 4, Jack is curious at what rate the beanstalk grew since day 2. Find the average rate of change so Jack can see the growth rate occurring.

(b) Interpret the above result for Jack.

(c) Let the two points lie on a tangent line to the actual height function of the beanstalk, $h(d)$, at $d = 3$ days. Find a tangent line and approximate for Jack the height of the beanstalk after 6 days.

(d) Jack continues to measure the height his beanstalk daily and puts together the following table:

d	0	2	4	6	8
$h(d)$	12.5	50	200	800	3200

Determine if $h(d)$ is a linear function. If it is, find the function. If it is not, determine what kind of function it is and find the function.

3. (20 points) Solve the following word problems.

   (a) Mary wants to build a rectangular fenced area next to the interestion of two rivers so her little lambs do not need her to bring water. The area she found looks like the below with dashed lines representing the riversides.

   If Mary has 1000 feet of fencing, what is the maximum area of the fenced area?

   (b) Mary realized the above area was a safety hazard for her lambs since they could fall in the river. Now she wants to construct a fenced rectangle of area 312500 square feet in the rainforest of Costa Rica, where the air is so humid it can act as their water, and away from any hazardous rivers (She does not realize the abundant hazards of the rainforest; dumb Mary). She does realize however that it will cost more for her to get fencing and is now concerned about this cost. For the first 3 sides, it costs $8 per foot and there is a discount on the last side of $2 per foot. What is minimum cost of the fencing?

4. (20 points) Bart Simpson is riding his skateboard from home to the Noiseland Arcade with an acceleration given by $a(t) = 12t^2 - 6t + 2$ in feet per square minute where t is in minutes. If it takes Bart 10 minutes to get to the arcade and Bart is not moving prior to his ride, find the following:

(a) Estimation of $\int_0^{10} v(t)\,dt$ using Left-hand Sums with $n = 2$ subdivisions (**include units**).

(b) Estimation of $\int_0^{10} v(t)\,dt$ using Right-hand Sums with $n = 2$ subdivisions (**include units**).

(c) Find a more accurate estimation using the two answers above (**include units**).

(d) Find a formula the distance when Bart has not left his house at time 0.

(e) Find the exact distance Bart travelled by integration and by evaluating the function found above at $t = 10$ (**include units**).

5. (20 points) Peter Parker and his Uncle Ben own a clothing store specializing in silk shirts called A Spider's Choice. The total cost for q shirts, in dollars, is given by $C(q) = 2000 + 500q$ and the total revenue for q shirts, in dollars, is given by $R(q) = 2000q - 10q^2$ (and yes; they are very expensive, high quality shirts).

(a) Find the profit when producing 15 shirts.

(b) Find the break-even point for the the business.

(c) Should the two produce the 16th shirt? Explain.

(d) Estimate the change in profit when producing the 16th shirt.

(e) At what quantity is revenue maximized?

(f) At what quantity is profit maximized?

6. (20 points) Draw a graph of each of the market models and label and find values for the following:

- equilibrium $(q*, p*)$.

- p-intercepts

- Consumer Surplus with value (use C.S.)

- Producer Surplus with value (use P.S.)

(a) Let the market be given by $D : \frac{1}{2}p + 4q = 15$ and $S : \frac{1}{3}p - q = \frac{8}{3}$.

(b) Come up with your own supply and demand curves, draw and find the value for each of the bullets.
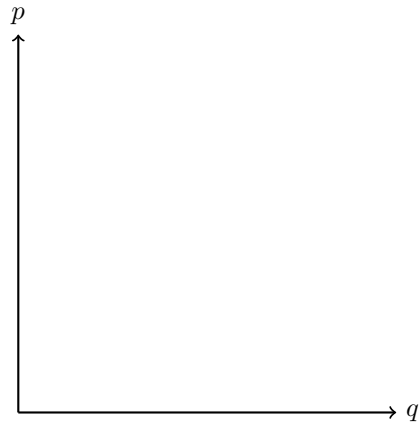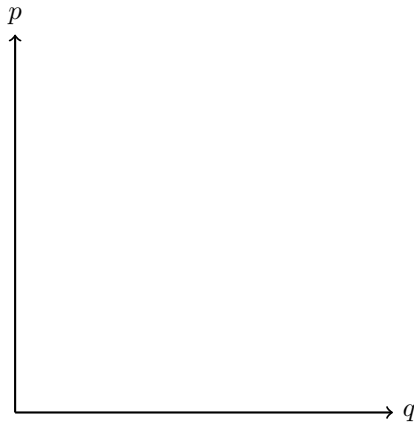
7. (20 points) Let Robert Downey Jr. invest \$3000 in an account compounded monthly at a rate of 3%. You decide to show Robert Downey Jr. how to REALLY invest money.

(a) If you were to invest the same amount as Robert Downey Jr. into an account with the same rate what is one characterstic the account could have that would ensure you make more than him? (Please only use things we learned in class)

(b) Write an equation for the amount in the account, P, in terms of years, t, for Robert Downey Jr.'s account as well as yours described above.

(c) Find the time at which the value of each account doubles. (**Show how you got your answer**)

(d) If the bank gave Mr Downey Jr. a better rate of 4%, how long would it take for your account to have more money than his?
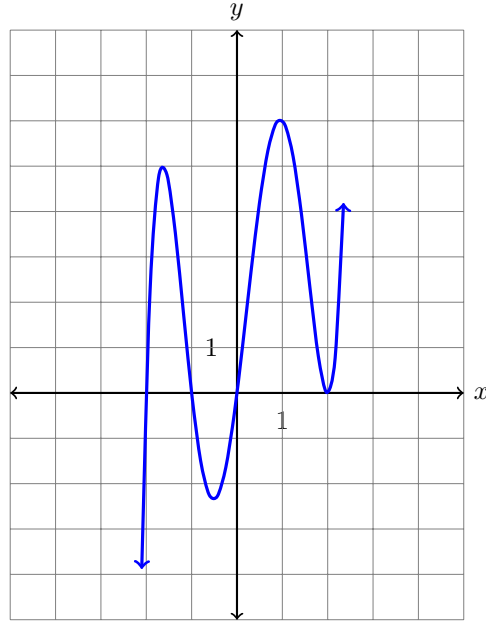
8. (20 points)

    (a) Given the following graph for revenue, sketch MR.
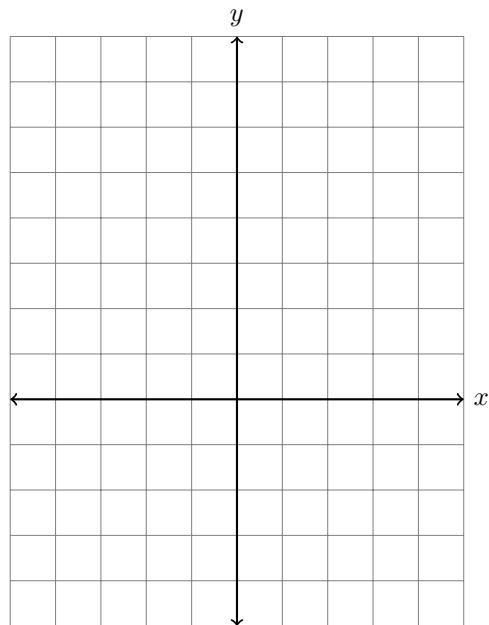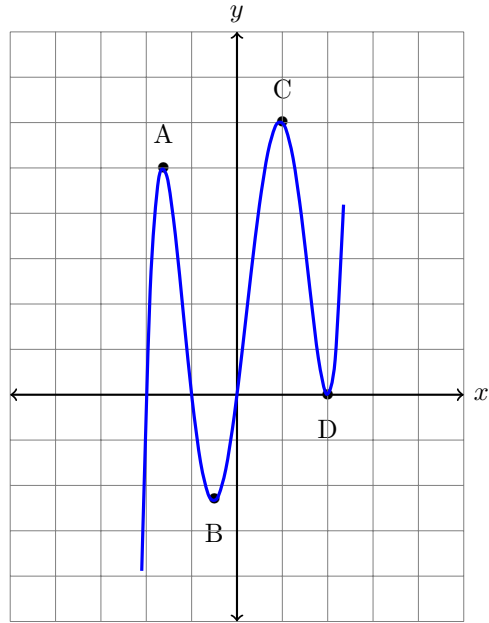
(b) Given the following **graph for** $f'(x)$, determine the following for $f(x)$:

- All the critical points
- Local maxima and local minima
- Increasing and decreasing intervals

Extra Credit (5 points): If you could choose a superpower what would it be? Now draw yourself using your superpower.