Instructions: This exam is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Remove any smartwatches and non-religious head-wear. Cheating of any kind will not be tolerated and will result in a grade of zero.**You must clear the memory on your calculator before beginning the exam.** Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. You have **24 hours** to finish this portion of the exam. Answer all questions to the best of your ability. Unless otherwise specified, you are required to **SHOW ALL YOUR WORK** to receive full credit. This portion of the exam has 80 possible points. You will be graded out of 75 points.

WRITE THIS PARAGRAPH ON WHAT YOU SUBMIT ALONG WITH A SIGNATURE AND DATE.

I, ____________________, will not under any circumstance use an online source, my notes, my peers, or any other resource besides my own knowledge and a calculator reset to factory settings to complete this exam. I will show all my work to demonstrate my knowledge on the topic.

Signature: ____________________ Date: ____________________

Questions	Possible	Score
Question 1	20	
Question 2	20	
Question 3	20	
Question 4	18	
Extra Credit	2	
Total		

1. (20 points) Ash is walking from Pallet Town to Viridian City with an acceleration given by $a(t) = 3t^2 - 5t + 2$ in yard per minute squared where t is in minutes. If it takes Ash 20 minutes to get to Viridian City and Ash is not moving prior to his trip, find the following:

   (a) Estimation of $\int_0^{20} v(t)\,dt$ using Left-hand Sums with $n = 5$ subdivisions (**include units**).

   Solution 1. $\Delta t = \dfrac{20-0}{5} = 4$

   $v(t) = \int a(t)\,dt = t^3 - \dfrac{5t^2}{2} + 2t + C$. *C=0 since Ash is not moving prior to his trip.*

   $LHS = 4(v(0) + v(4) + v(8) + v(12) + v(16)) = 4(0 + 32 + 368 + 1392 + 3488) = 21120$ *yards*

   (b) Estimation of $\int_0^{20} v(t)\,dt$ using Right-hand Sums with $n = 5$ subdivisions (**include units**).

   Solution 2. $\Delta t = \dfrac{20-0}{5} = 4$

   $v(t) = \int a(t)\,dt = t^3 - \dfrac{5t^2}{2} + 2t + C$. *C=0 since Ash is not moving prior to his trip.*

   $RHS = 4(v(4) + v(8) + v(12) + v(16) + v(20)) = 4(32 + 368 + 1392 + 3488 + 7040) = 49280$ *yards*

   (c) Find a more accurate estimation using the two answers above (**include units**).

   Solution 3. $\dfrac{LHS + RHS}{2} = \dfrac{21120 + 49280}{2} = 35200$ *yards*

   (d) Find a formula the distance when Ash is still in Pallet Town at time 0.

   Solution 4. $d(t) = \int v(t)\,dt = \int (t^3 - \dfrac{5t^2}{2} + 2t)\,dt = \dfrac{t^4}{4} - \dfrac{5t^3}{6} + t^2 + C$. *C=0 since Ash is still in Pallet Town at time 0.*

   (e) Find the exact distance Ash travelled to get to Viridian City (**include units**). Hint: Find the distance function and reread the information at the beginning of the problem.

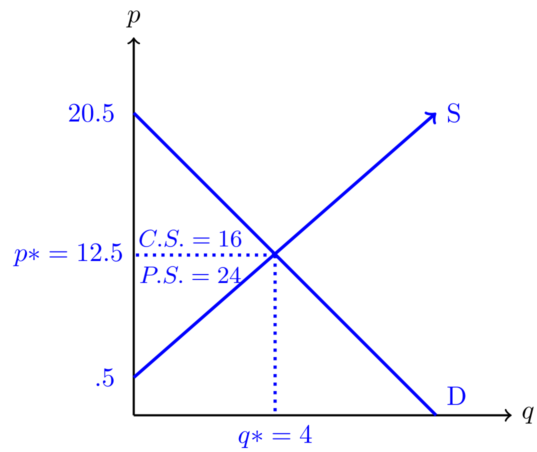
   Solution 5. $d(t) = \dfrac{t^4}{4} - \dfrac{5t^3}{6} + t^2 \Rightarrow d(20) = \dfrac{20^4}{4} - \dfrac{5(20)^3}{6} + 20^2 = 33733.33...$ *yards. This can ablso be found by using the definite integral from 0 to 20 of* $v(t)$*.*

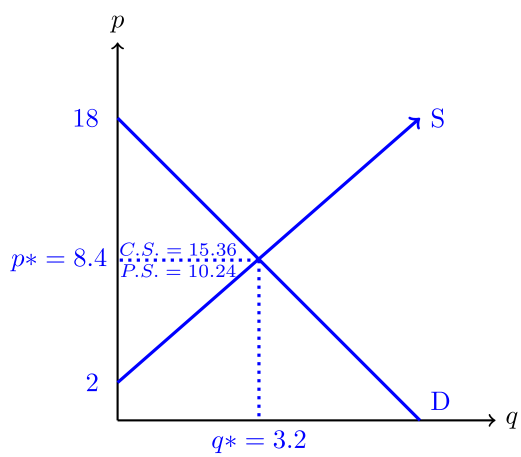
2. (20 points) Draw a graph of each of the market models. Then find (values) and label the following:

- equilibrium $(q*, p*)$.
- p-intercepts
- Consumer Surplus with value (use C.S.)
- Producer Surplus with value (use P.S.)

(a) Let the market be given by $D(q) = -2q + \frac{41}{2}$ and $S(q) = 3q + \frac{1}{2}$.

(b) Let the market be given by $D : \frac{1}{2}p + \frac{3}{2}q = 9$ and $S : \frac{1}{2}p - q = 1$.





3. (20 points) Mario and Luigi are competing to save Princess Peach. Whoever gets to Princess Peach first gets to stop being a plumber and becomes king (As we all know this never lasts long...). Let table 1 represent the velocity of Mario $v_m(t)$ in meters per minute with t in minutes and table 2 represent the velocity of Luigi $v_l(t)$ in meters per minute with t in minutes.

   (a) Table 1.

t	20	23	26	29	32	35	38
$v_m(t)$	62	72	87	92	7	666	333

   (b) Table 2.

t	15	$15\frac{1}{2}$	16	$16\frac{1}{2}$	17	$17\frac{1}{2}$	18
$v_l(t)$	554	881	1280	1177	2019	1234	1

We may assume they both arrived at the same time since they started their adventures at different times (AKA We may assume they weren't moving in the time before the first blocks of the table). Since they arrived at the same time, the winner of the throne will be decided by who took the shortest path to Princess Peach. Answer the following questions to determine the winner of the throne.

   (a) (3 points each) Find left-hand sums for the distance travelled by Mario and Luigi (Label each answer as Mario or Luigi with units).

   Solution 6. $M : LHS = 3(62 + 72 + 87 + 92 + 7 + 666) = 2958$

   $L : LHS = .5(554 + 881 + 1280 + 1177 + 2019 + 1234) = 3572.5$

   (b) (3 points each) Find right-hand sums for the distance travelled by Mario and Luigi (Label each answer as Mario or Luigi with units).

   Solution 7. $M : RHS = 3(72 + 87 + 92 + 7 + 666 + 333) = 3771$

   $L : RHS = .5(881 + 1280 + 1177 + 2019 + 1234 + 1) = 3296$

   (c) (1 point each) Using the above, come up with the best estimation we know for each and use them to determine who gets to take the throne and who goes back to being a plumber. (Label each answer as Mario or Luigi with units).

   Solution 8. $M : \frac{LHS + RHS}{2} = \frac{2958 + 3771}{2} = 3364.5$

   $L : \frac{LHS + RHS}{2} = \frac{3572.5 + 3296}{2} = 3434.25$

   Mario wins the thrown because he traveled the shortest distance to get to Princess Peach.

4. (18 points)

(a) Find the area, when $x \geq 0$, between the two curves:

$$f(x) = x^3 + 1 \qquad\qquad g(x) = x^2 + 2x + 1$$

Solution 9. *Let* $x = 1$*.* $f(1) = 2$ *and* $g(1) = 4$*. Thus,* g *is bigger when* $x \geq 0$*.*

$x^3 + 1 = x^2 + 2x + 1 \Rightarrow x^3 - x^2 - 2x = 0 \Rightarrow x(x^2 - x - 2) = 0 \Rightarrow x(x-2)(x+1) = 0$
Thus, we will use $x = 0$ *and* $x = 2$*.*

$$\int_0^2 (x^2 + 2x + 1) - (x^3 + 1)\, dx = \int_0^2 (-x^3 + x^2 + 2x)\, dx = -\frac{x^4}{4} + \frac{x^3}{3} + x^2 \Big|_0^2$$
$$= -\frac{2^4}{4} + \frac{2^3}{3} + 2^2 - (0) = 2.66...$$

(b) Find the area, when $x \leq 0$, between the two curves:

$$f(x) = x^3 + 1 \qquad\qquad g(x) = x^2 + 2x + 1$$

Solution 10. *By the above, we should choose* x *such that* $-1 < x < 0$ *.Let* $x = -.5$*.* $f(-.5) = .875$ *and* $g(-.5) = .25$*. Thus,* f *is bigger when* $x \leq 0$*.*

$x^3 + 1 = x^2 + 2x + 1 \Rightarrow x^3 - x^2 - 2x = 0 \Rightarrow x(x^2 - x - 2) = 0 \Rightarrow x(x-2)(x+1) = 0$
Thus, we will use $x = 0$ *and* $x = -1$*.*

$$\int_{-1}^0 (x^3 + 1) - (x^2 + 2x + 1)\, dx = \int_0^2 (x^3 - x^2 - 2x)\, dx = \frac{x^4}{4} - \frac{x^3}{3} - x^2 \Big|_{-1}^0$$
$$= (0) - \left(\frac{(-1)^4}{4} - \frac{(-1)^3}{3} - (-1)^2\right) = .4166...$$

5. Extra Credit (2 points): Draw your spirit animal with an explanation as to why it is your spirit animal.