Instructions: This exam is an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Cheating of any kind will not be tolerated and will result in a grade of zero.**You must clear the memory on your calculator before beginning the exam.** Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. Write your answers on the test or all on seperate pieces of paper. You have until Friday, March 27th to finish the exam and submit it to Blackboard. Answer all questions to the best of your ability. Unless otherwise specified, you are required to **SHOW ALL YOUR WORK** to receive full credit. The exam has 108 possible points. You will be graded out of 100 points.

WRITE THIS PARAGRAPH ON WHAT YOU SUBMIT ALONG WITH A SIGNATURE AND DATE.

I, ____________________, will not under any circumstance use an online source, my peers, or any other resource besides my own knowledge, my notes, and a calculator reset to factory settings to complete this exam. I will show all my work to demonstrate my knowledge on the topic. If I do use notes, I will not consider previous quizzes, homeworks, and worksheets as notes. I will also not use any work I happened to do in my notes on these assignments/worksheets as this is unfair and is considered cheating.

Signature: ____________________ Date: ____________________

Questions	Possible	Score		Possible	Score
Question 1	12		Question 6	8	
Question 2	18		Question 7	12	
Question 3	8		Question 8	12	
Question 4	16		Question 9	8	
Question 5	12		Extra Credit	4	
				Total	

1. (12 points) For this problem you need only give your solution unless otherwise stated by the question. Part (g) and (c) are worth 2 points. All others are worth 1 point.

   (a) If $f'(a) > 0$ then $f(x)$ is increasing at a.

   (b) What is the nth deriviative of $f(x) = x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ where a_i is a constant for all i? $n! = n \cdot (n-1) \cdots 1$ at a.

   (c) Give two other functions that are the same as $d''(t)$ where $d(t)$ is distance. $v'(t) = a(t)$

   (d) The second derivative at a, $f''(a)$, determines if $f(x)$ is concave up or down at a.

   (e) If $C'(100) = 5.2$ and $R'(100) = 6$, determine the change in profit for producing and selling the 101$^{\text{st}}$ unit.
   $\pi(100) = R'(100) - C'(100) = .8$

   (f) Marginal revenue at q quantity is the slope of the tangent line at q of the renvenue function.

   (g) Let $f'(1) = 0$, $f''(0) = 0$, and $f'(-1) = 0$. Also, let $f'(x) < 0$ for $x < -1$ and $x > 1$, $f'(x) > 0$ for $-1 < x < 1$, $f''(x) < 0$ for $x < 0$, and $f''(x) > 0$ for $x > 0$. Determine the local maxima(s), local minima(s), and inflection point(s). **Clearly label which is which**.
   Min: $x = -1$, Max $x = 1$, inflection $x = 0$

   (h) $\frac{d}{dx}\Big(\ln(x)\Big) = \frac{1}{x}$

   (i) (T/F) $\frac{d}{dx}\left[\frac{h(x)}{k(x)}\right] = \frac{h'(x)k(x) - k'(x)h(x)}{k(x)}$ F

   (j) (T/F) $\frac{d}{dx}\Big[h(k(x))\Big] = h'(k'(x))$ F

2. (3 points each) Find the first and second derivative for the following functions:

(a) $f(x) = x^3 - \frac{3}{2}x^2 + 13^2$

Solution 1. $f'(x) = 3x^2 - 3x$
$f''(x) = 6x - 3$

(b) $f(x) = 3\ln(x) + 2x^2 - 8\mathrm{e}^x + 2^x$

Solution 2. $f'(x) = \frac{3}{x} + 4x - 8e^x + \ln(2) \cdot 2^x$
$f''(x) = \frac{-3}{x^2} + 4 - 8e^x + (\ln(2))^2 \cdot 2^x$

(c) $f(x) = \mathrm{e}^{2x^2}$

Solution 3. $f'(x) = e^{2x^2} \cdot 4x$
$f''(x) = 4e^{2x^2} + 16x^2e^{2x^2}$

(d) $f(x) = 2x\ln(x^2)$

Solution 4. $f'(x) = 2\ln(x^2) + 2x \cdot \frac{2x}{x^2} = 2\ln(x^2) + 4$
$f''(x) = \frac{4x}{x^2} = \frac{4}{x}$

(e) $f(x) = \dfrac{12}{5\sqrt[3]{x}}$

Solution 5. $f(x) = \frac{12}{5}x^{-1/3}$
$f'(x) = -\frac{4}{5}x^{-4/3}$
$f''(x) = \frac{16}{15}x^{-7/3}$

(f) $f(x) = \dfrac{e^x}{x}$

Solution 6. $f'(x) = \frac{xe^x - e^x}{x^2} = \frac{e^x}{x} - \frac{e^x}{x^2}$
$f''(x) = \frac{xe^x - e^x}{x^2} - \frac{x^2e^x - 2xe^x}{x^4} = \frac{x^3e^x - x^2e^x - x^2e^x + 2xe^x}{x^4}$

3. (8 points) John, Jacob, and Jingleheimer are all brothers who own The Schmidt Brothers Sandwich Shop. The total cost for q sandwiches, in dollars, is given by $C(q) = \ln\left(\frac{q}{100}\right) + 20$ and the total revenue for q sandwiches, in dollars, is given by $R(q) = e^{\sqrt{q}}$. (Both are for $0 \leq q \leq 100$.)

(a) Find the profit when producing 20 sandwiches.

Solution 7. $\pi(q) = R(q) - C(q) = e^{\sqrt{q}} - (\ln\left(\frac{q}{100}\right) + 20)$
$\pi(20) = e^{2\sqrt{5}} + \ln(5) - 20 = 69.152...$

(b) Should the brothers produce the 21st sandwich? Explain without graphs.

Solution 8. $\pi'(q) = R'(q) - C'(q) = \frac{1}{2}q^{-1/2}e^{\sqrt{q}} - \frac{1}{q}$
$\pi'(20) = \frac{1}{4\sqrt{5}}e^{2\sqrt{5}} - \frac{1}{20} = 9.73766$

4. (16 points) Let $h(x) = 2^{g(x)f(x)}$ and $k(x) = \frac{f(x)+g(x)}{f(x)-g(x)}$ such that $f(1) = 2$, $g(1) = 7$, $g'(1) = 3$, and $f'(1) = 1$. Find $h'(1)$ and $k'(1)$.

Solution 9. $h'(x) = \ln(2) \cdot 2^{g(x)f(x)} \cdot (f'(x)g(x) + g'(x)f(x))$
$h'(1) = \ln(2) \cdot 2^{7\cdot 2} \cdot (1(7) + 3(2)) = \ln(2) \cdot 2^{7\cdot 2} \cdot 13 = 147634.8043$

$k'(x) = \frac{(f'(x)+g'(x))(f(x)-g(x))-(f'(x)-g'(x))(f(x)+g(x))}{(f(x)-g(x))^2}$
$k'(1) = \frac{(f'(1)+g'(1))(f(1)-g(1))-(f'(1)-g'(1))(f(1)+g(1))}{(f(1)-g(1))^2} = \frac{4(-5)-(-2)9}{(-5)^2} = \frac{-2}{25}$

5. (12 points) Given the graph below where $A = (a, f(a))$, $B = (b, f(b))$, $C = (c, f(c))$, $D = (d, f(d))$, and $E = (e, f(e))$, find the following:

(a) Determine the local minima and local maxima of the graph. Label each.

Solution 10. *Min: A, E*
Max: C

(b) Find the intervals for which the function is increasing and the intervals for which the function is decreasing and label which are increasing and which are decreasing (Note: Use open brackets for these intervals).

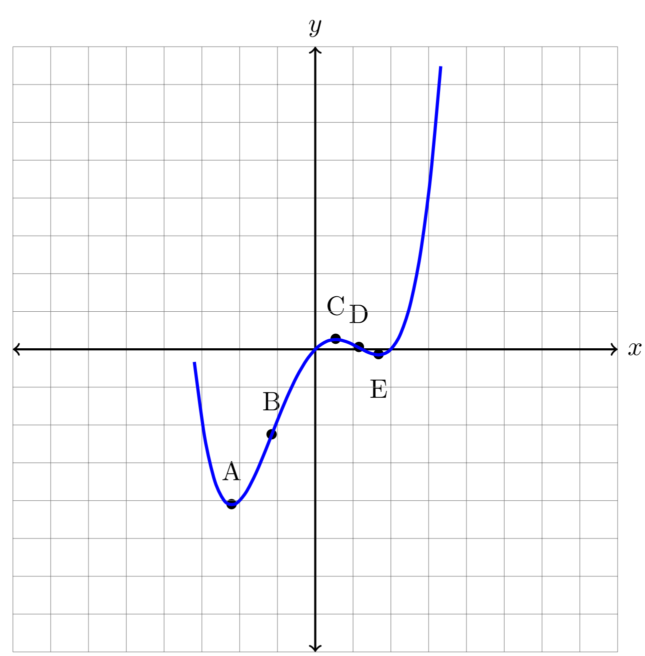
Solution 11. *Inc: A to C, E to infinity*
Dec: neg infinity to A, C to E

(c) Find the intervals for which the function is concave up and the intervals for which the function is concave down and label which are concave up and which are concave down (Note: Use open brackets for these intervals).

Solution 12. *Up: neg infinity to B, D to infinity*
Down: B to D

(d) Find all inflection points on the graph.

Solution 13. *B and D*



6. (8 points) Let $f(x)$ be a function. Let (2,5) and (-2,1) be points on the tangent line of $f(x)$ at $x = 2$. Let $g(x) = x^2 + \frac{x}{x^2+1}$.

   (a) (2) Find $f'(2)$ and give an equation for the tangent line to $f(x)$ at $x = 2$.

   Solution 14. $f'(2) \approx \frac{5-1}{2+2} = 1$
   Tangent line: $y - 5 = x - 2$ *or* $y - 1 = x + 2 \rightarrow y = x + 3$

   (b) (2) Using the above, approximate $f(5)$.

   Solution 15. $f(5) = 5 + 3 = 8$.

   (c) (4) Find the equation of the line tangent to $g(x)$ at $x = 1$.

   Solution 16. $g'(x) = 2x + \frac{x^2+1-2x^2}{(x^2+1)^2}$
   $g'(1) = 2$
   $g(1) = 1 + \frac{1}{2} = \frac{3}{2}$
   Thus, the tangent line is $y - \frac{3}{2} = 2(x - 1)$

7. (12 points) Solve the following world problems.

(a) Me and Julio want to build a rectangular fenced area down by the schoolyard. The rectangular area will share a fence with the schoolyard and thus one side is already fenced. If we have 5000 feet of fencing, what is the maximum area of the fenced area?

Solution 17. $2x + y = 5000$ *and* $A = xy$
$\Rightarrow y = 5000 - 2x$
$A = 5000x - 2x^2$
$A' = 5000 - 4x = 0$
$x = 1250$
$\Rightarrow 5000 - 2(1250) = 2500 = y$
$A = 2500(1250) = 3125000 ft^2$

(b) Julio and I have changed our minds. Now we want to construct the fenced area in a rectangle of area 5000 square feet in the middle of nowhere We are not the best farmers... We are now more concerned about the cost of fencing. For the first 3 sides, it costs \$50 per foot and there is a discount on the last side of \$25 per foot. What is minimum cost of the fencing?

Solution 18. $5000 = xy$ *and* $C = (2x + y)50 + 25y = 100x + 75y$
$x = \frac{5000}{y}$
$C = \frac{500000}{y} + 75y$
$C' = -\frac{500000}{y^2} + 75 = 0$
$y^2 = \frac{500000}{75}$
$y = 81.64965...$
$x = \frac{5000}{81.64965...} = 61.23724...$
$C = \$12247.4487...$

Extra Credit (4 points): Draw a picture of both of the fenced areas and a reason why we moved locations! (Be super creative!)

8. (12 points)

(a) Let the function, $f(x)$, have the folllowing derivative:

$$f'(x) = 2x^2(x+3)^3(x-1)(x-5)$$

Fill in the following table with (+) or (-) and determine the possible local minimum(s) and maximum(s) values. **SHOW ALL WORK.**

Table 1.

	$(x+3)^3$	$2x^2$	$(x-1)$	$(x-5)$	$f'(x)$
$x < -3$	-	+	-	-	-
$-3 < x < 0$	+	+	-	-	+
$0 < x < 1$	+	+	-	-	+
$1 < x < 5$	+	+	+	-	-
$x > 5$	+	+	+	+	+

Solution 19. *Use test points to fill in the table as work.*

Min: $x = -3, 5$

Max: $x = 1$

(b) Let the function, $f(x)$, have the folllowing derivative:

$$f''(x) = x(x+1/2)^2(x-1)^2(x-4)$$

Fill in the following table and determine the intervals of concavity. **SHOW ALL WORK.**

Table 2.

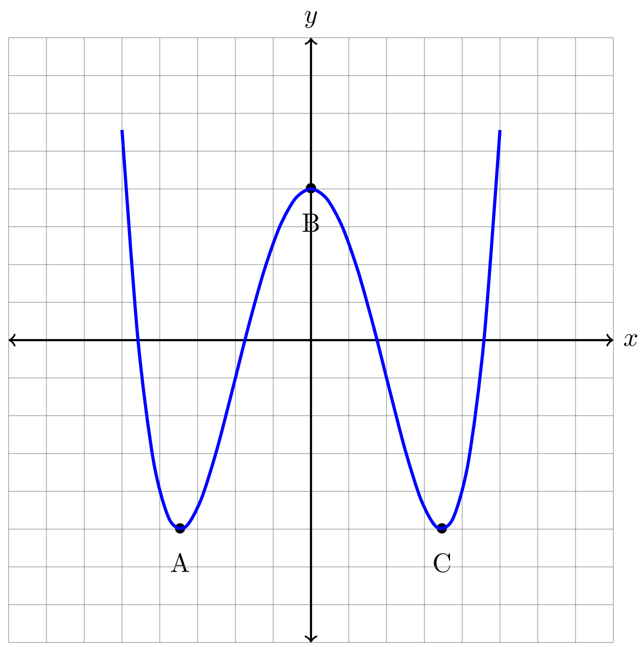
	$(x+1/2)^2$	x	$(x-1)^2$	$(x-4)$	$f''(x)$
$x < -1/2$	+	-	+	-	+
$-1/2 < x < 0$	+	-	+	-	+
$0 < x < 1$	+	+	+	-	-
$1 < x < 4$	+	+	+	-	-
$x > 4$	+	+	+	+	+

Solution 20. *Up: neg infinity to 0 and 4 to infinity*

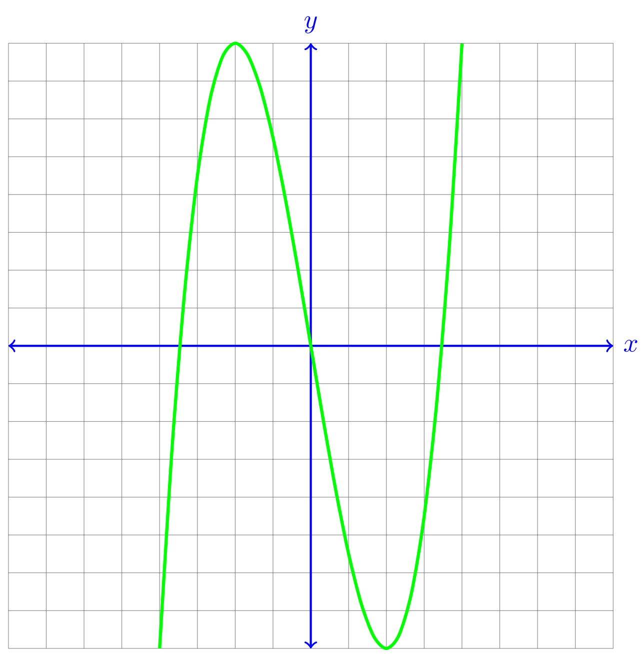
Down: 0 to 4

9. (8 points)

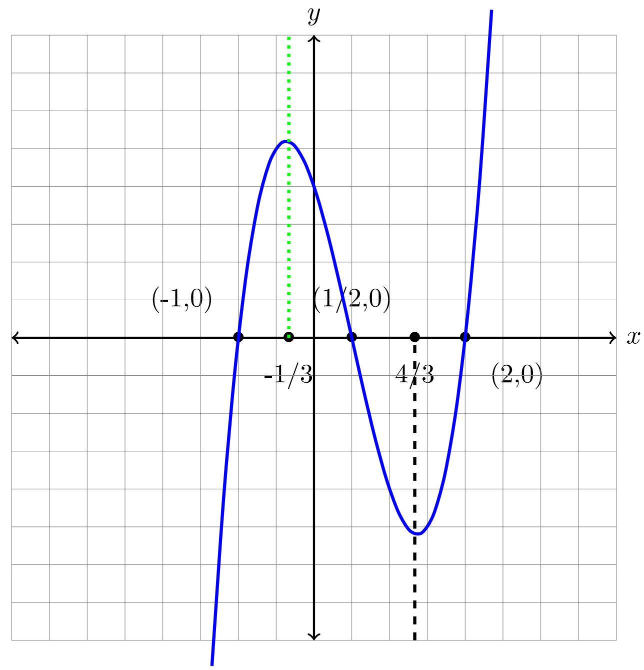
(a) Given the following graph for $f(x)$, sketch the derivative, $f'(x)$.



Solution 21. *Here is a graph of the derivative.*



(b) Given the following graph for $f'(x)$, give information about $f(x)$ (This includes increasing/decreasing, maxima minimum, etc.).



Solution 22. *Inc: -1 to 1/2, 2 to infinity*
Dec: neg infinity to -1, 1/2 to 2
Min: $x = -1, 2$
Max: $x = 1/2$
Inflection points and concavity intervals are also easily seen.